Building reference point based composite indicators with different compensability levels: some reflections

Samira El Gibari, José Manuel Cabello, Trinidad Gómez and Francisco Ruiz

Department of Applied Economics (Mathematics)
University of Málaga
Spain

UNIVERSIDAD DE MÁLAGA

Table of contents

Introduction

- A composite indicator (*CI*) is an aggregate of all dimensions, objectives, individual indicators and variables used (Nardo *et al.*, 2008).
- The *CI* should ideally measure multidimensional concepts which cannot be captured by a single indicator (Nardo *et al.*, 2005).
- A number of methodologies have been developed to construct CIs $\rightarrow$ see Nardo *et al.* (2005, 2008) and El Gibari *et al.* (2018).
- Important aspect:
  - The **compensation** issue.

Background

- The reference point method (Wierzbicki, 1980): Efficient solutions by using one reference level (desired).
- The double reference point scheme (Wierzbicki *et al.*, 2000): Efficient solutions and objective rankings with two reference levels.
- Construction of composite indicators with different compensation degrees (Ruiz *et al.*, 2011).
  1. Only two reference levels (Reservation and Aspiration levels).
  2. A fixed scale.

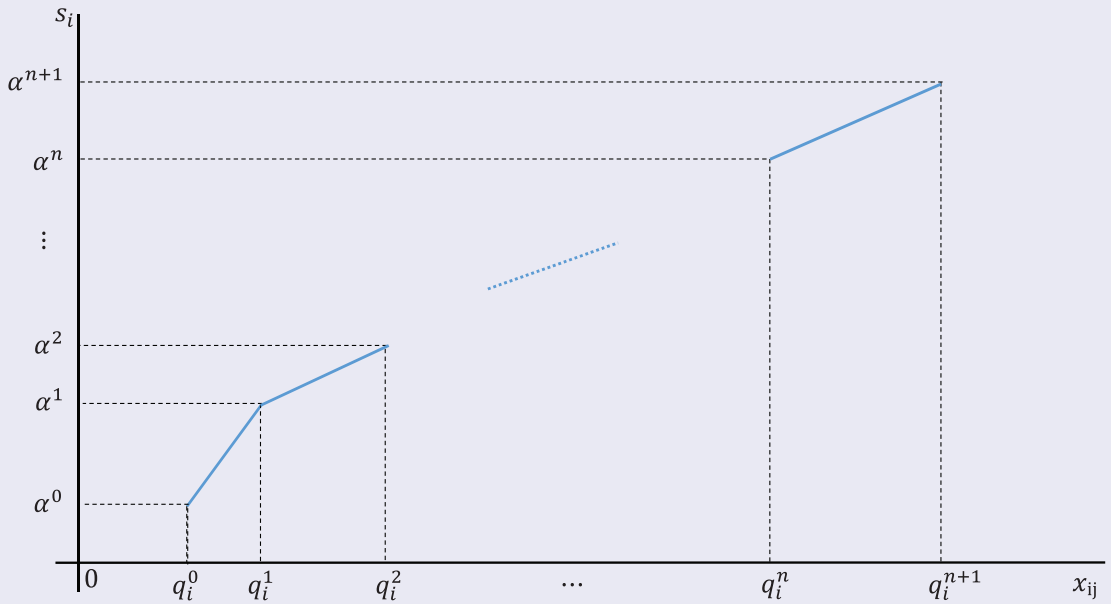
A generalization of the double reference point method

The **Multiple Reference Point-Weak and Strong Composite Indicator** (MRP-WSCI) approach (Ruiz *et al.*, 2020)

The *MRP-WSCI* approach

A generalization of the double reference point method

- Any number of reference levels (n) can be established.
- A common measurement scale: $\alpha^t (t = 0, \ldots, n+1)$.
- Achievement functions for each indicator:



$$s_i(x_i^j, q_i) = \alpha^{t-1} + \frac{\alpha^t - \alpha^{t-1}}{q_i^t - q_i^{t-1}}(x_i^j - q_i^{t-1}) \quad \text{if } x_i^j \in [q_i^{t-1}, q_i^t],$$
$$(t = 1, \ldots, n+1)$$

The *MRP-WSCI* approach

Different composite indicators

Full compensation:

1. Weak Composite Indicator (*WCI*) $\rightarrow$ A measure of overall performance.

$$WCI^j = \sum_{i=1}^{I} \mu_i^w s_i(x_i^j, q_i)$$

   - Properties of classical weighted means.
   - $\mu_i^w \rightarrow$ Normalized weights add up to 1:

$$\mu_i^w = \frac{\mu_i}{\sum_{k=1}^{I} \mu_k}$$

The *MRP-WSCI* approach

Different composite indicators

Zero compensation:

2. Without weights $\rightarrow$ Unweighted Strong Composite Indicator (*USCI*).

$$USCI^j = \min_{i=1,\ldots,I}\{s_i(x_i^j, q_i)\}$$

- A measure of the worst performance of the unit.
- The minimum value of the achievement functions for the unit.

The *MRP-WSCI* approach

Different composite indicators

Zero compensation:

3. Consideration of weights → Strong Composite Indicator (*SCI*).

$$SCI^j = \min_{i=1,\ldots,I}\{\bar{s}_i(x_i^j, q_i)\}$$

- A measure of the worst values, relativized by the weights of the indicators → Higher weights produce worse results.
- $\mu_i^s \rightarrow$ Normalized weights, where the greatest weight takes value 1:

$$\mu_i^s = \frac{\mu_i}{\max_{k=1,\ldots,I}\{\mu_k\}}$$

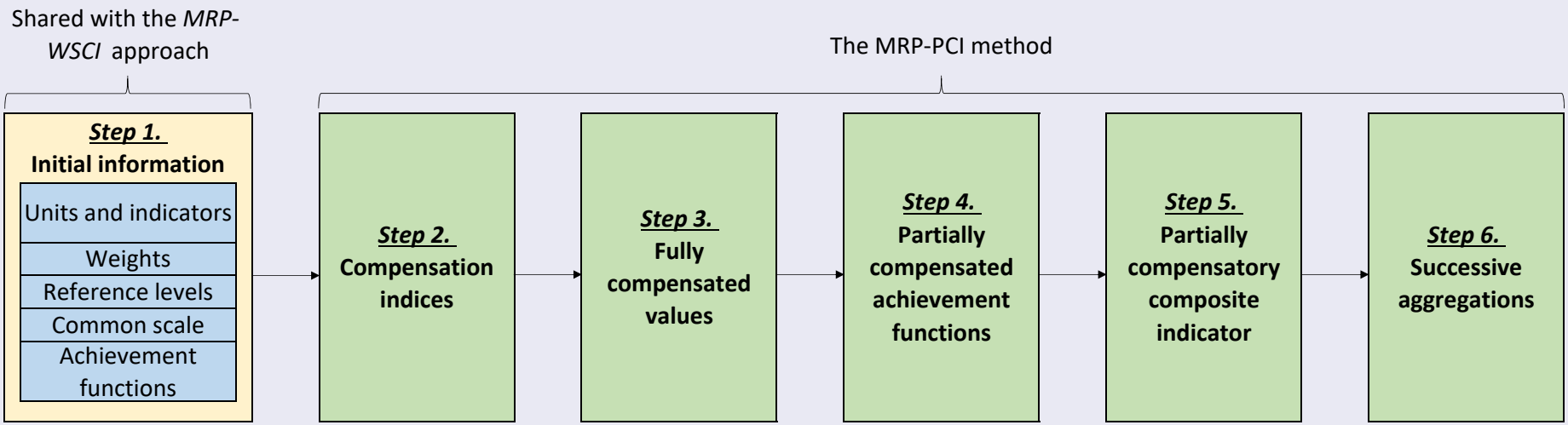
- Corrected achievement functions:

$$\bar{s}_i(x_{ij}, q_i) = \begin{cases} \alpha^t + (s_i(x_{ij}, q_i) - \alpha^t)\mu_i^s, & \text{if } s_i(x_{ij}, q_i) \in (\alpha^{t-1}, \alpha^t], \\ & (t = 1, \ldots, n+1), \\ \alpha^1 + (\alpha^0 - \alpha^1)\mu_i^s, & \text{if } s_i(x_{ij}, q_i) = \alpha^0. \end{cases}$$

The *MRP-PCI* approach

The *MRP-PCI* Partially Compensatory Indicator

- The decision maker can decide which indicators cannot be compensated, which can, and to which extent (Ruiz and Cabello, 2021).
- Step-Wise Description:



The *MRP-PCI* approach

Step-Wise Description

2. Compensation indices $\rightarrow$ The decision maker can provide a compensation index, $\lambda_i \in [0, 1]$, for each indicator i.
3. Fully compensated values $\rightarrow$ a_{ij} is the weighted average of s_{ij} and the rest of achievement function values that are at least as good as s_{ij}.

$$a_{ij} = \frac{\sum_{k \in I_{ij}} \mu_k s_{kj}}{\sum_{k \in I_{ij}} \mu_k},$$

where I_{ij} is the subset of indicators that take a value better or equal to indicator i for unit j:

$$I_{ij} = \{k \in \{1, \ldots, I\} \mid s_{kj} \geq s_{ij}\}.$$

The *MRP-PCI* approach

Step-Wise Description

4. Partially compensated achievement functions $\rightarrow s_{ij}^c$ is a link between s_{ij} and a_{ij}, according to λ_i.

$$s_{ij}^c = s_{ij} + (a_{ij} - s_{ij})\lambda_i \rightarrow \begin{cases} If\, \lambda_i = 0 \rightarrow s_{ij}^c = s_{ij}, \\ If\, \lambda_i = 1 \rightarrow s_{ij}^c = a_{ij}. \end{cases}$$

5. Partially compensatory composite indicator.

$$PCI_j = \min_{i=1,\ldots,I} \{s_{ij}^c\}.$$

6. Successive aggregations $\rightarrow$ The *MRP-WSCI* and *MRP-PCI* composite indicators can be used as achievement functions in a multi-stage aggregation process.

The *MRP-WSCI* & *MRP-PCI* approaches

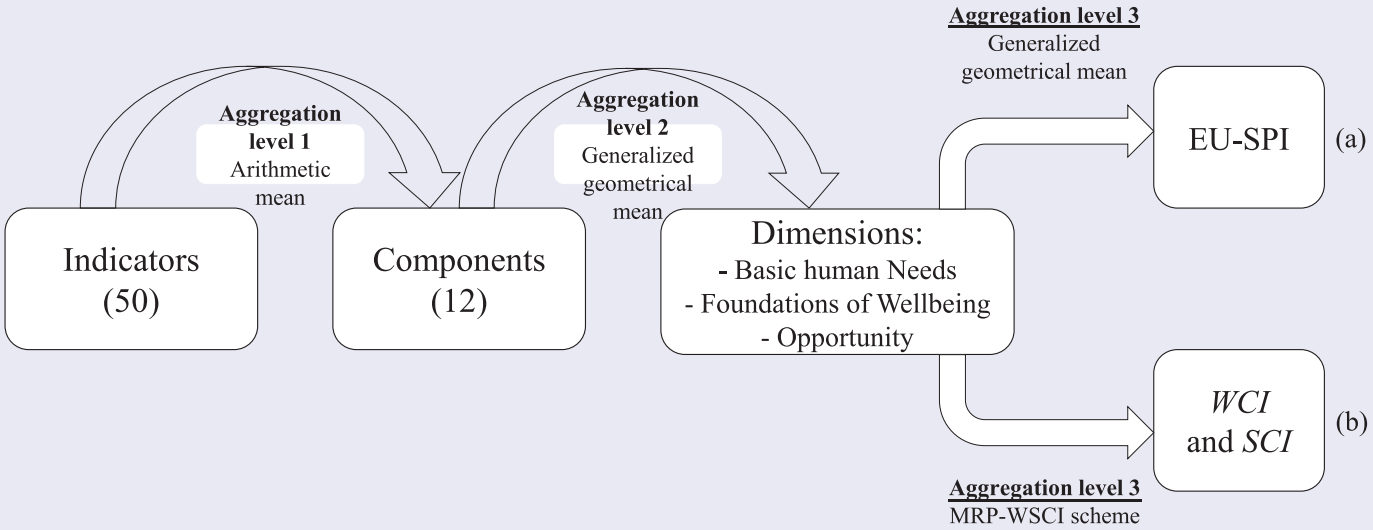
Theoretical properties

- The *MRP-WSCI* indicators satisfy desirable properties for composite indicators $\rightarrow$ Existence and uniqueness, symmetry, transitivity, monotonicity, invariance and exhaustivity (Blancas *et al.*, 2010).
- The *MRP-PCI*$_j$ indicator always lies between the *WCI*$_j$ and the *SCI*$_j$.
  - *WCI*$_j$ $\rightarrow$ A particular case of *PCI*$_j$ when all the compensation indices are equal to 1.
  - *SCI*$_j$ $\rightarrow$ A particular case of *PCI*$_j$ when all the compensation indices are equal to 0.

The *MRP-WSCI* approach

Case study

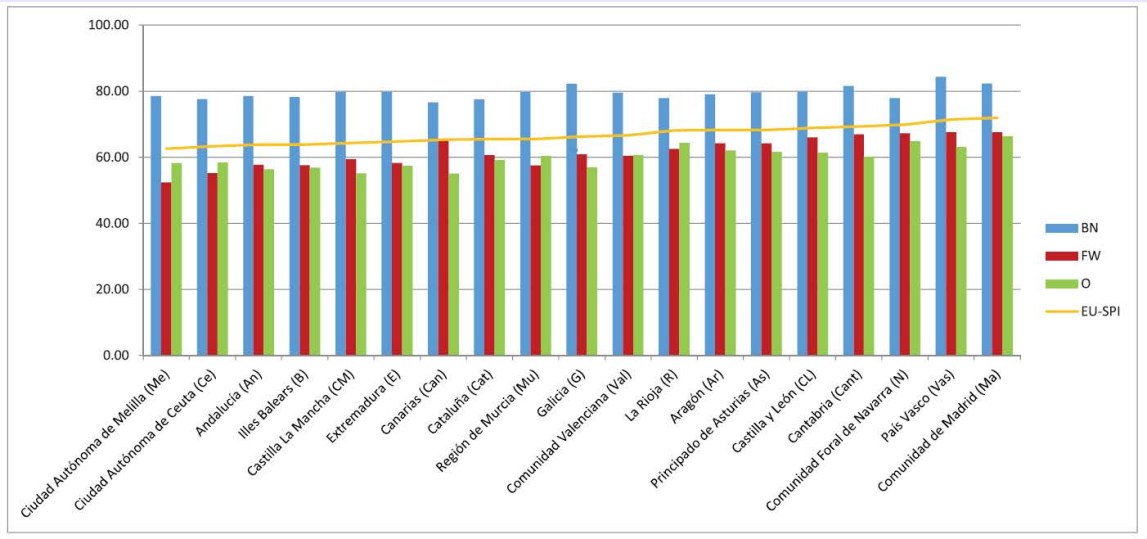
- Construction of the EU-Regional Social Progress Index (EU-SPI), in the Spanish context.



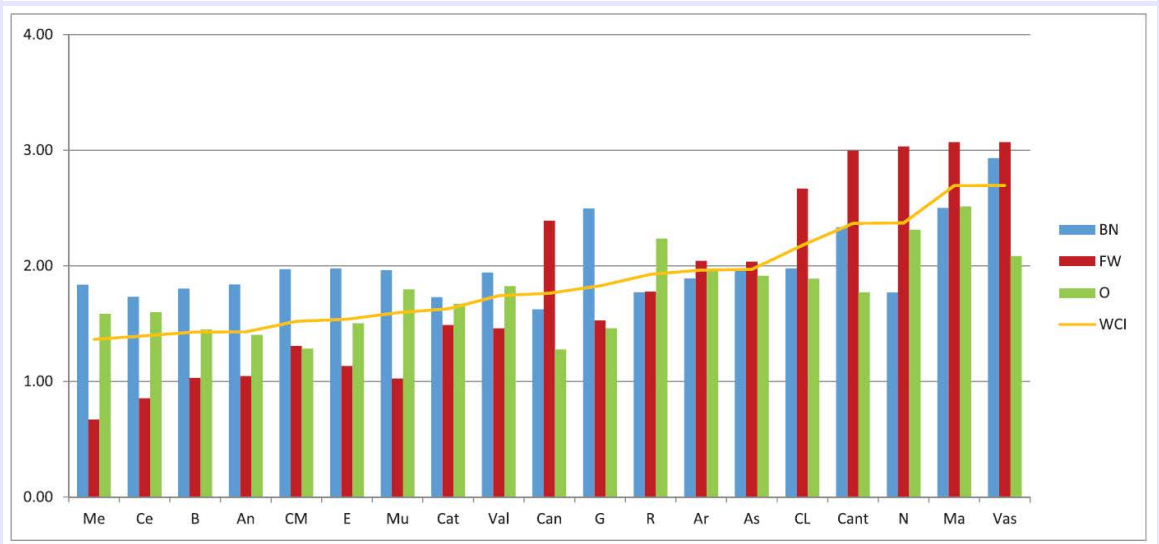
- Reference levels: minimum, maximum and percentiles 25, 50 and 75 of all the European regions.
- Common scale: $\alpha^0 = 0$, $\alpha^1 = 1$, $\alpha^2 = 2$, $\alpha^3 = 3$, $\alpha^4 = 4$.
- Same weights.

The *MRP-WSCI* approach

Case study



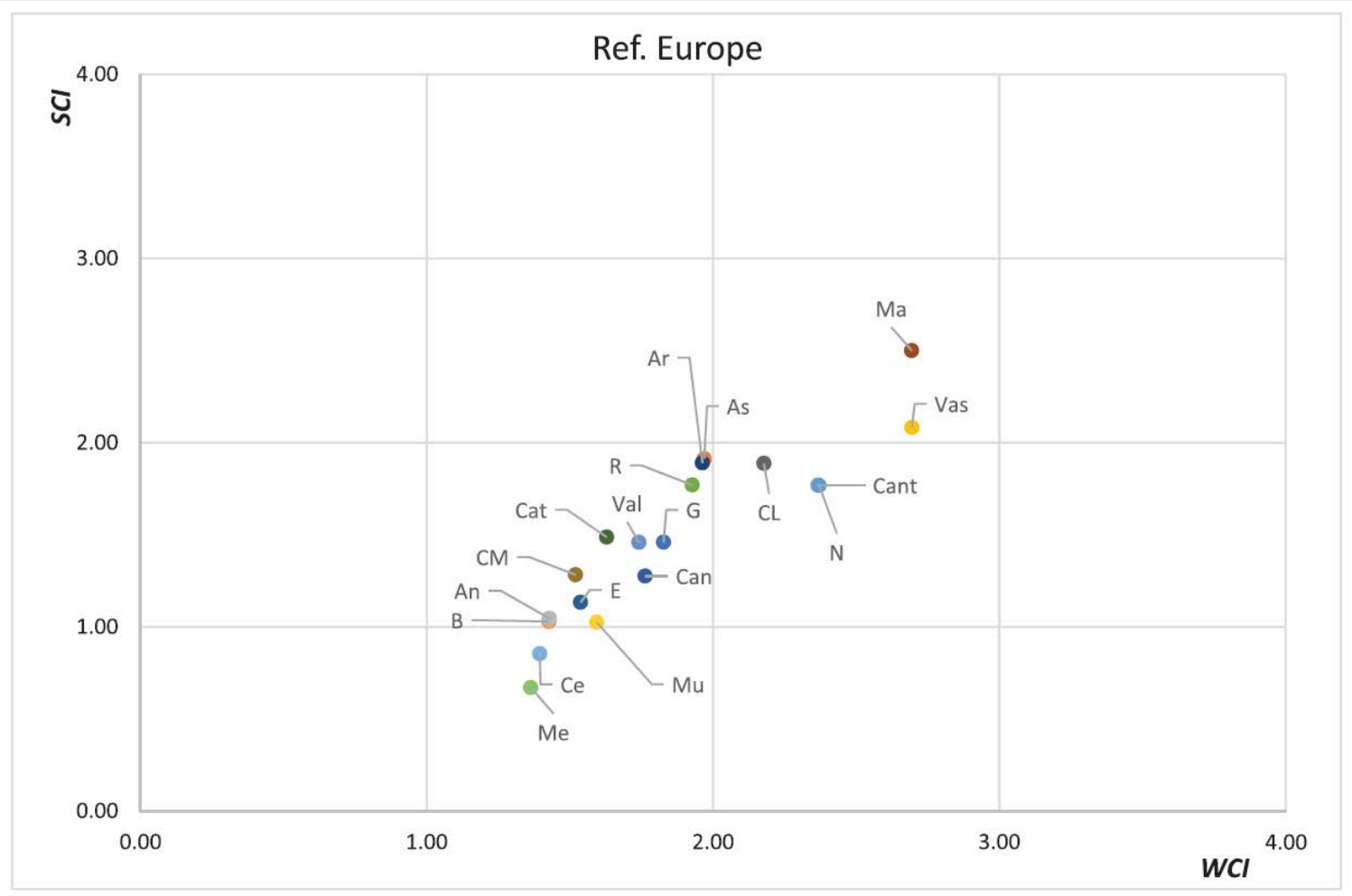
EU-SPI



WCI

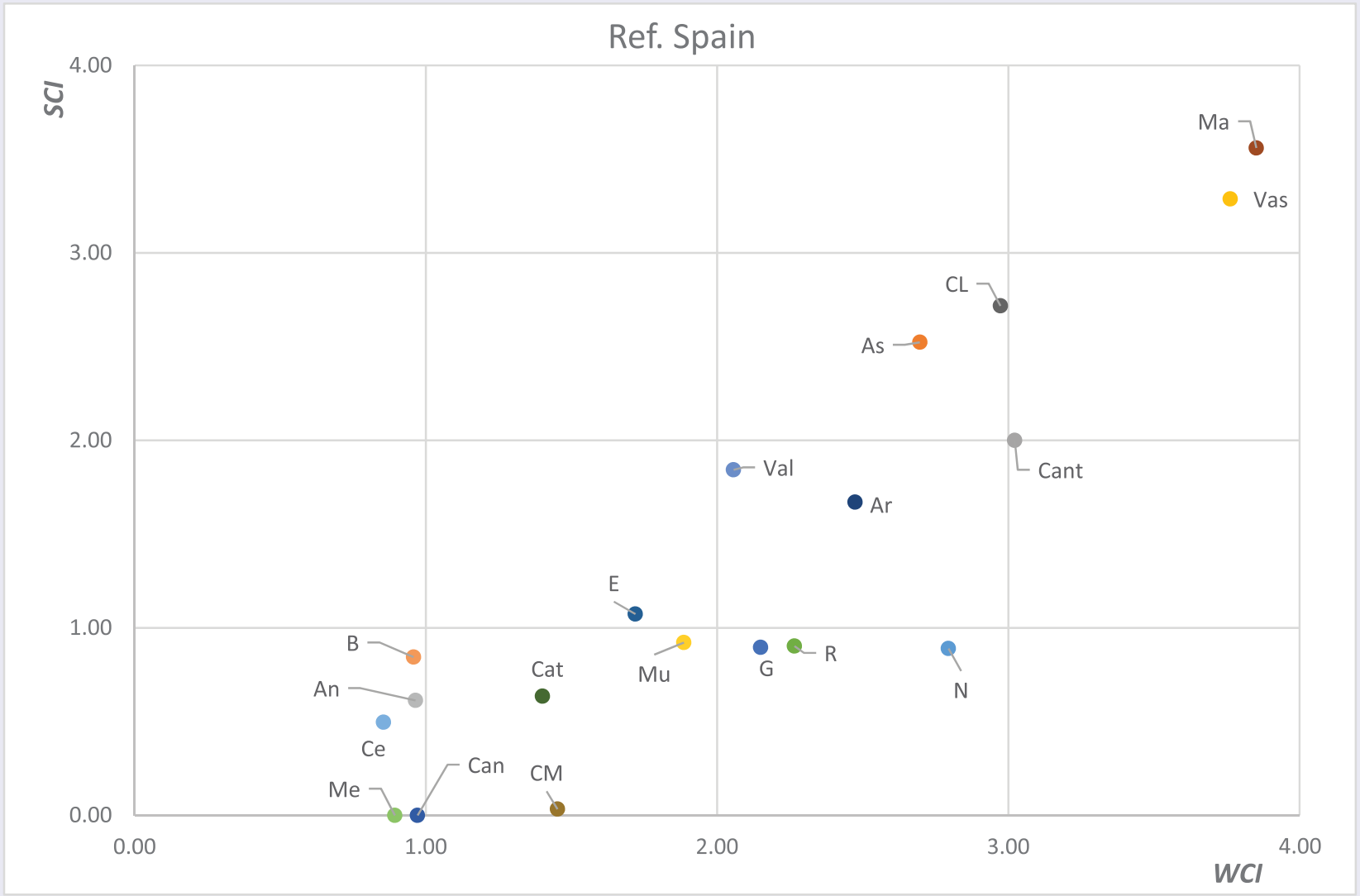
The *MRP-WSCI* approach

Case study: Representation of *WCI* and *SCI*



The *MRP-WSCI* approach

Case study: Changing the reference levels



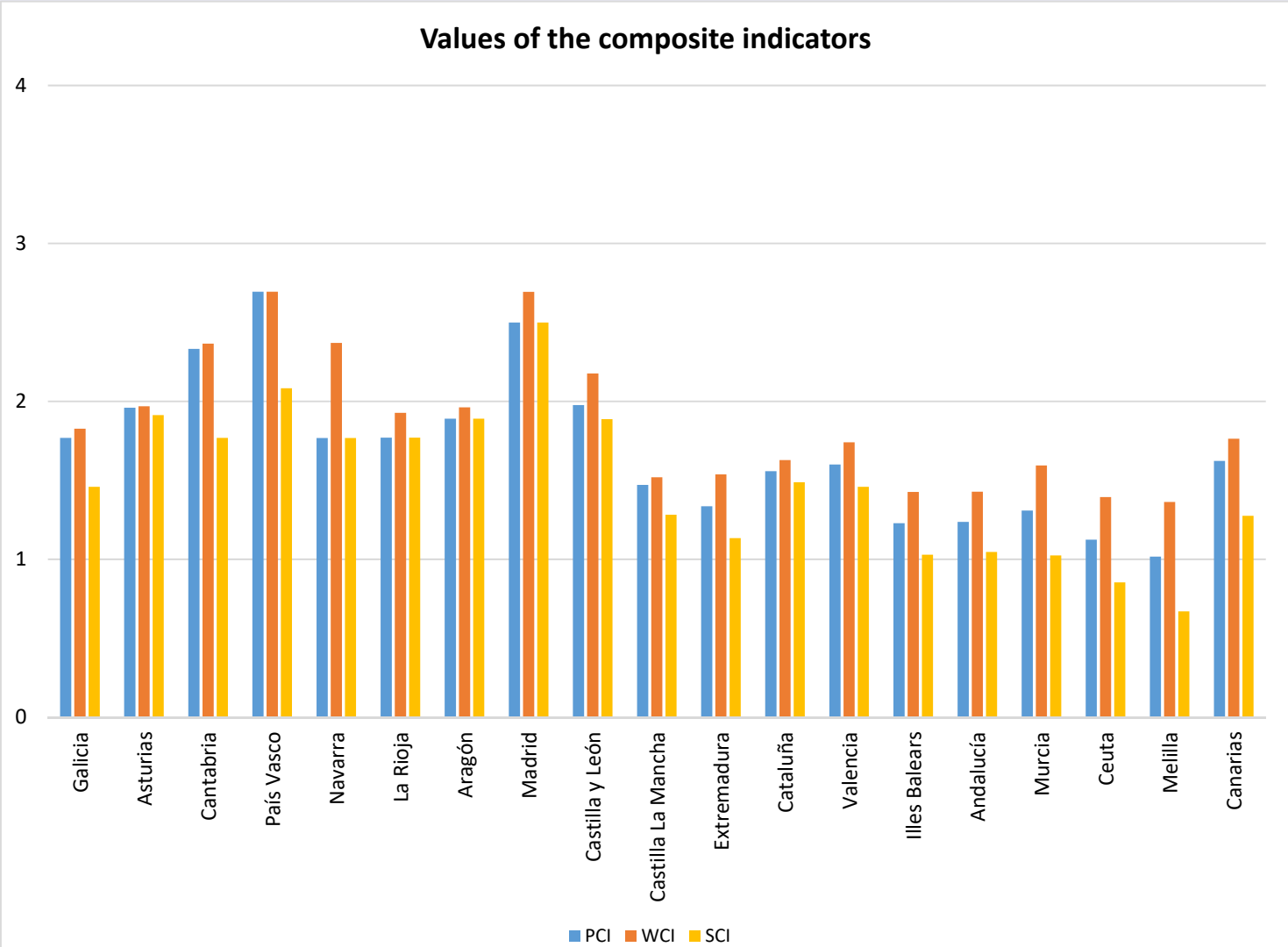
The *MRP-PCI* approach

Case study: A hypothetical example of the *PCI*

- Three value scale:
  - $\lambda_1 = 0 \rightarrow$ No compensation: a bad value of the *Basic Human Needs* dimension cannot be compensated in any way.
  - $\lambda_2 = 0.5 \rightarrow$ Middle compensation: a bad value of the *Foundations of Wellbeing* dimension can be partially (half) compensated by better values in other dimensions.
  - $\lambda_3 = 1 \rightarrow$ Full compensation: a bad value of the *Opportunity* dimension can be completely compensated by better values in the other two dimensions.

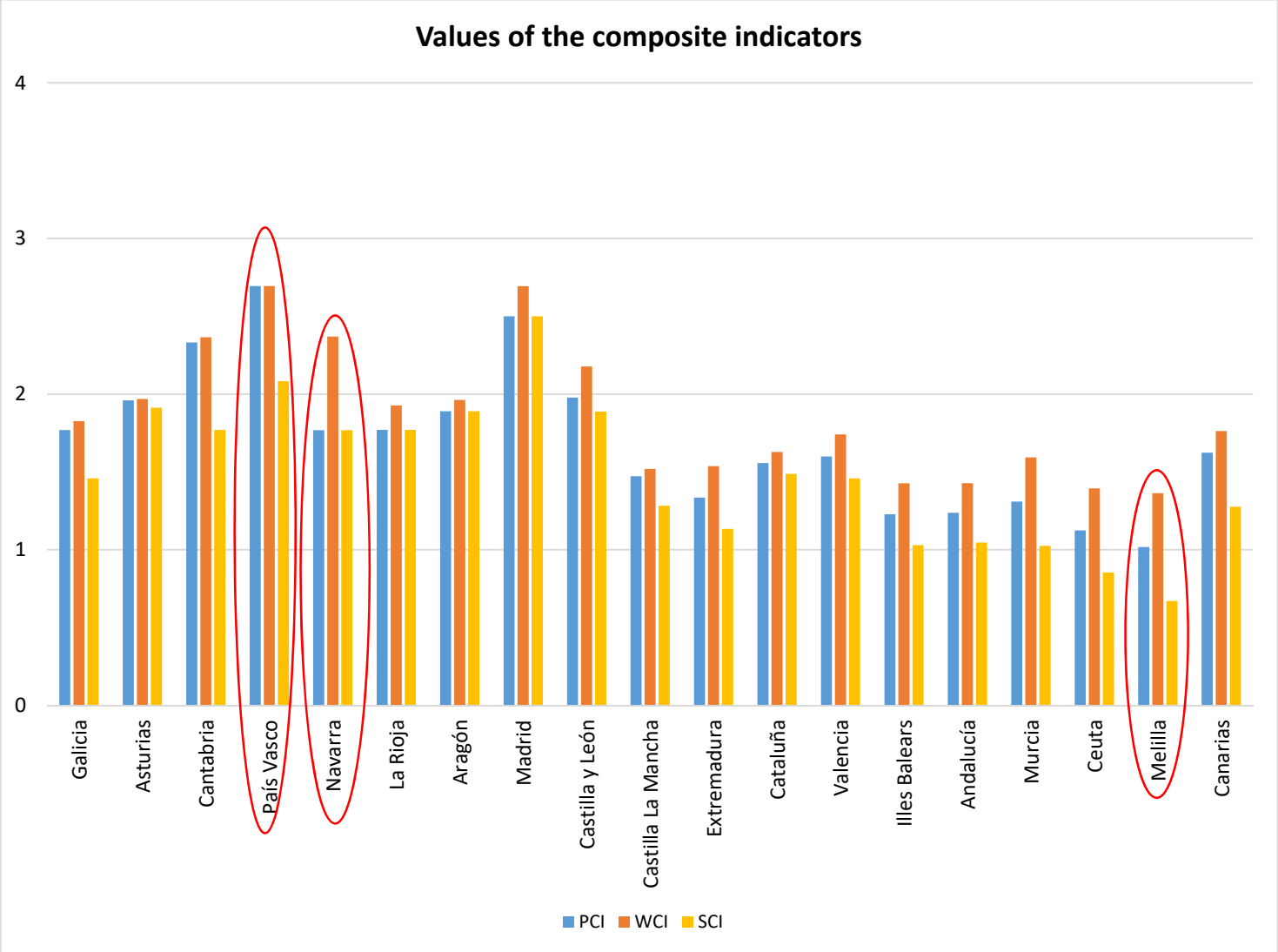
The *MRP-PCI* approach

Case study



The *MRP-PCI* approach

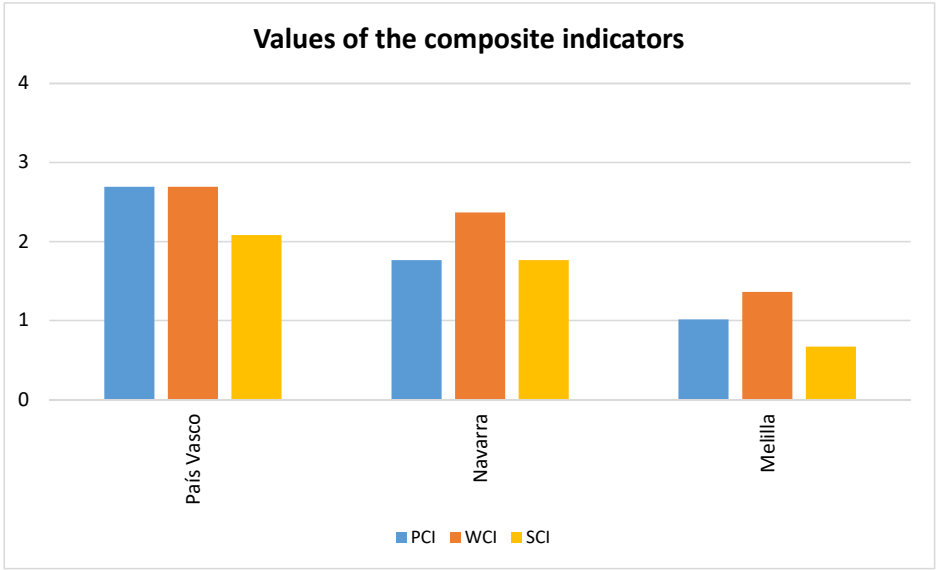
Case study



The *MRP-PCI* approach

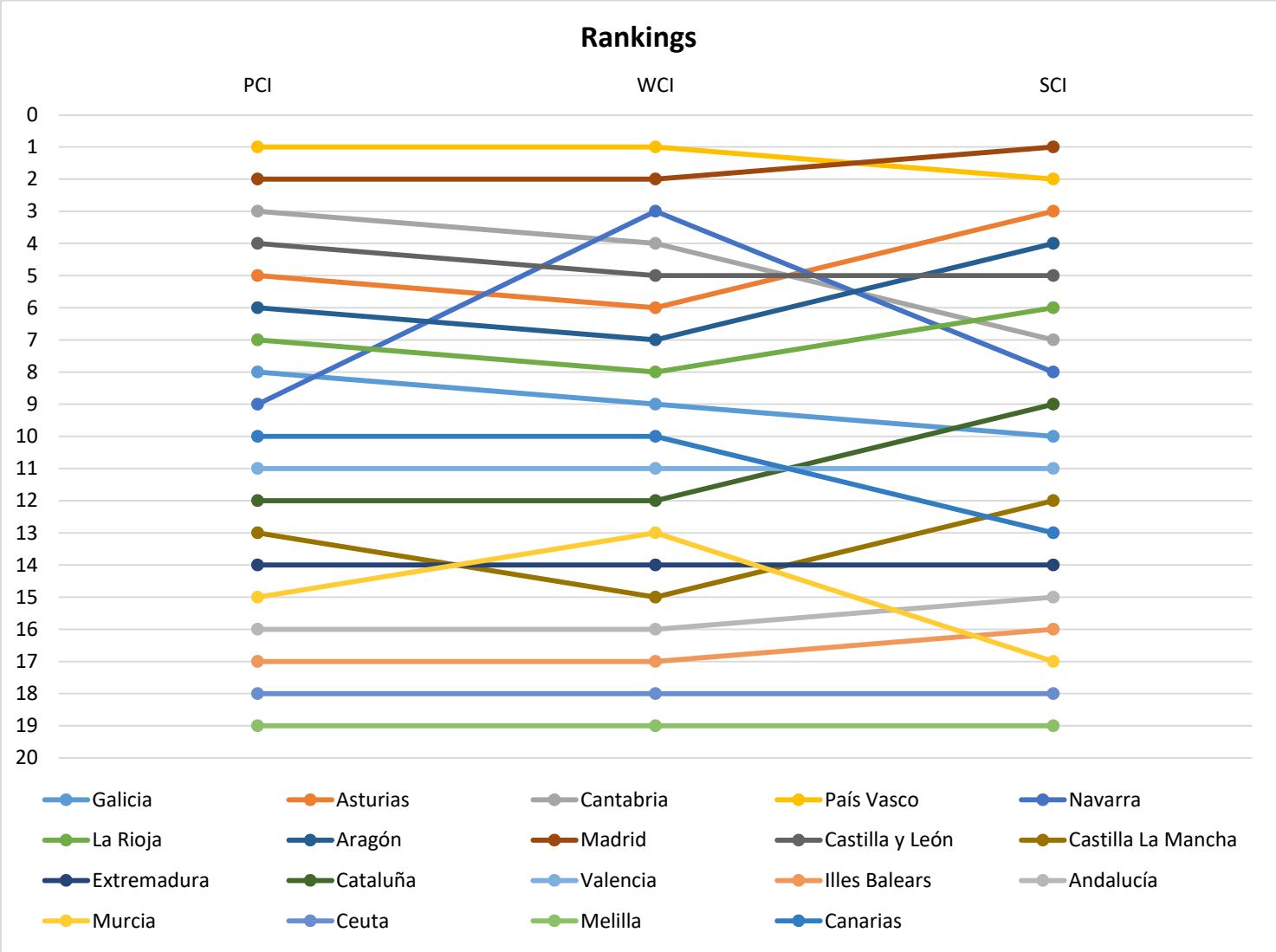
Case study

	Achievement functions			Fully compensated values (aij)			Partially compensated achievement functions (scij)			MRP-PCI	MRP-WSCI	
Region	**BN**	**FW**	**O**	**BN**	**FW**	**O**	**BN**	**FW**	**O**	**PCI**	**WCI**	**SCI**
País Vasco	2.93	3.07	2.08	3.00	3.07	2.69	2.93	3.07	2.69	2.69	2.69	2.08
Navarra	1.77	3.03	2.31	2.37	3.03	2.67	1.77	3.03	2.67	1.77	2.37	1.77
Melilla	1.84	0.67	1.58	1.84	1.36	1.71	1.84	1.02	1.71	1.02	1.36	0.67



The *MRP-PCI* approach

Case study



Conclusions

- The construction and use of composite indicators → Less loss of information about the individual indicators.
- The *MRP-WSCI* indicators satisfy desirable properties for composite indicators.
- The interpretation of the *MRP-WSCI* is intuitive and easy for the decision maker → Usefulness of the results.
  - Different compensation degrees.
  - Twofold analysis:
    - Overall performance.
    - Alert signs → Possible improvement areas.

Conclusions

- The joint consideration of the compensatory and non-compensatory scenarios $\rightarrow$ A richer information.
- The *MRP-PCI* method $\rightarrow$ A high modelling flexibility:
  - The decision maker can use different compensation indices for each indicator.
- The Compensation indices $\rightarrow$ An intuitive and easy-to-interpret way.
- The *MRP-WSCI* composite indicators $\rightarrow$ Particular cases of the *MRP-PCI* approach.

Further applications

1. Sustainability:
   - Luque, M. (2011). An application of reference point techniques to the calculation of synthetic sustainability indicators. Journal of the Operational Research Society, 62: 189–197.
   - Cabello, J.M., Navarro, E., Prieto, F., Rodríguez, B., Ruiz, F. (2014). Multicriteria Development of Synthetic Indicators of the Environmental Profile of the Spanish Regions. Ecological Indicators, 39: 10–23.
   - Cabello, J.M., Navarro, E., Rodríguez, B., Thiel, D., Ruiz, F. (2019). Dual weak–strong sustainability synthetic indicators using a double reference point scheme: the case of Andalucía, Spain. Operational Research, An International Journal, 19: 757–782.

Further applications

1. Sustainability:
   - Cabello, J.M., Navarro, E., Thiel, D., Rodríguez, B., Ruiz, F. (2021). Assessing environmental sustainability by the double reference point methodology: the case of the provinces of Andalusia (Spain). International Journal of Sustainable Development & World Ecology, 28: 4–17.
   - Ruiz, F. and Cabello, J. M. (2021). MRP-PCI: A Multiple Reference Point Based Partially Composite Indicators for Sustainability Assessment. Sustainability, 13, 1261.

Further applications

2. Carrying capacity of tourist destinations:
   - Ruiz, F., Cabello, J.M., Navarro, E., Tejada, M., Almeida, F., Cabello, J.M., Cortés, R., Delgado, J., Fernández, F., Gutiérrez, G., Luque, M., Málvarez, G., Marcenaro, O., Navas, F., Ruiz, F., Ruiz, J., Solís, F. (2012). Carrying capacity assessment for tourist destinations. Methodology for the creation of synthetic indicators applied in a coastal area. Tourism Management, 33: 1337–1346.
3. Social responsibility of funds:
   - Cabello, J.M., Ruiz, F., Pérez, B., Méndez, P. (2014). Synthetic Indicators of Mutual Funds' Environmental Responsibility: An Application of the Reference Point Method. European Journal of Operational Research, 236: 313–325.

Further applications

4. Bankruptcy prediction:
   - Ouenniche, J., Bouslah, K., Cabello, J.M., Ruiz, F. (2018). A New Classifier based on The Reference Point Method with Application in Bankruptcy Prediction. Journal of the Operational Research Society, 69(10): 1653–1660.
5. Ease of doing business:
   - Ruiz, F., Cabello, J.M., Pérez, B. (2018). Building Ease-of-Doing-Business Synthetic Indicators using a Double Reference Point Approach. Technological Forecasting & Social Change, 131: 130–140.
6. Regional innovation:
   - García-Bernabéu, A., Cabello, J.M., Ruiz, F. (2020). A Multi-Criteria Reference Point Based Approach for Assessing Regional Innovation Performance in Spain. Mathematics, 8(5): 797.

Further applications

7. Economic freedom:
   - Cabello, J.M., Ruiz, F., Pérez-Gladish, B. (2021). An Alternative Aggregation Process for Composite Indexes: An Application to the Heritage Foundation Economic Freedom Index. Social Indicators Research, 153: 443–467.
8. Higher education:
   - El Gibari, S. (2020). Análisis multicriterio del rendimiento del Sistema Universitario Público Españñol: Hay vida más allá de los rankings. Revista Electrónica de Comunicaciones y Trabajos de ASEPUMA (Rect),21, 119–150.
   - El Gibari, S., Gómez, T., and Ruiz, F. (2018). Evaluating university performance using reference point based composite indicators.Journal of Informetrics, 12, 1235–1250.

Further applications

9. End of childhood index:
   - El Gibari, S., Cabello, J. M., Gómez, T., and Ruiz, F. (2021). Composite indicators as decision making tools: the joint use of compensatory and non-compensatory schemes.International Journal of Information Technology & Decision Making, To appear.

Future research lines

- The time dimension.
- The weighting procedures $\rightarrow$ *SCI*.

Thank you

UNIVERSIDAD DE MÁLAGA

European Commission

References

Blancas, F. J., Caballero, R., González, M., Lozano-Oyola, M., and Pérez, F. (2010). Goal programming synthetic indicators: An application for sustainable tourism in Andalusian coastal counties. *Ecological Economics*, **69**, 2158–2172.

El Gibari, S., Gómez, T., and Ruiz, F. (2018). Evaluating university performance using reference point based composite indicators. *Journal of Informetrics*, **12**, 1235–1250.

Nardo, M., Saisana, M., Saltelli, A., and Tarantola, S. (2005). Tools for Composite Indicators Building. Technical report, European Commission, Ispra(Italy).

Nardo, M., Saisana, M., Saltelli, A., Tarantola, S., Giovannini, E., and Hoffman, A. (2008). Handbook on Constructing Composite Indicators. Methodology and user guide. Technical report, OECD, Paris.

Ruiz, F. and Cabello, J. M. (2021). MRP-PCI: A Multiple Reference Point Based Partially Composite Indicators for Sustainability Assessment. *Sustainability*, **13**, 1261.

Ruiz, F., Cabello, J. M., and Luque, M. (2011). An application of reference point techniques to the calculation of synthetic sustainability indicators. *Journal of the Operational Research Society*, **62**, 189–197.

Ruiz, F., El Gibari, S., Cabello, J. M., and Gómez, T. (2020). MRP-WSCI: Multiple reference point based weak and strong composite indicators. *Omega*, **95**, 102060.

Wierzbicki, A. P. (1980). The use of reference objectives in multiobjective optimization. In G. Fandel and T. Gal, editors, *Lecture Notes in Economics and Mathematical Systems, vol. 177*, pages 468–486, Berlin. Springer-Verlag.

Wierzbicki, A. P., Makowski, M., and Wessels, J., editors (2000). *Model-Based Decision Support Methodology with Environmental Applications*. Kluwer Academic Publishers, Dordrecht.