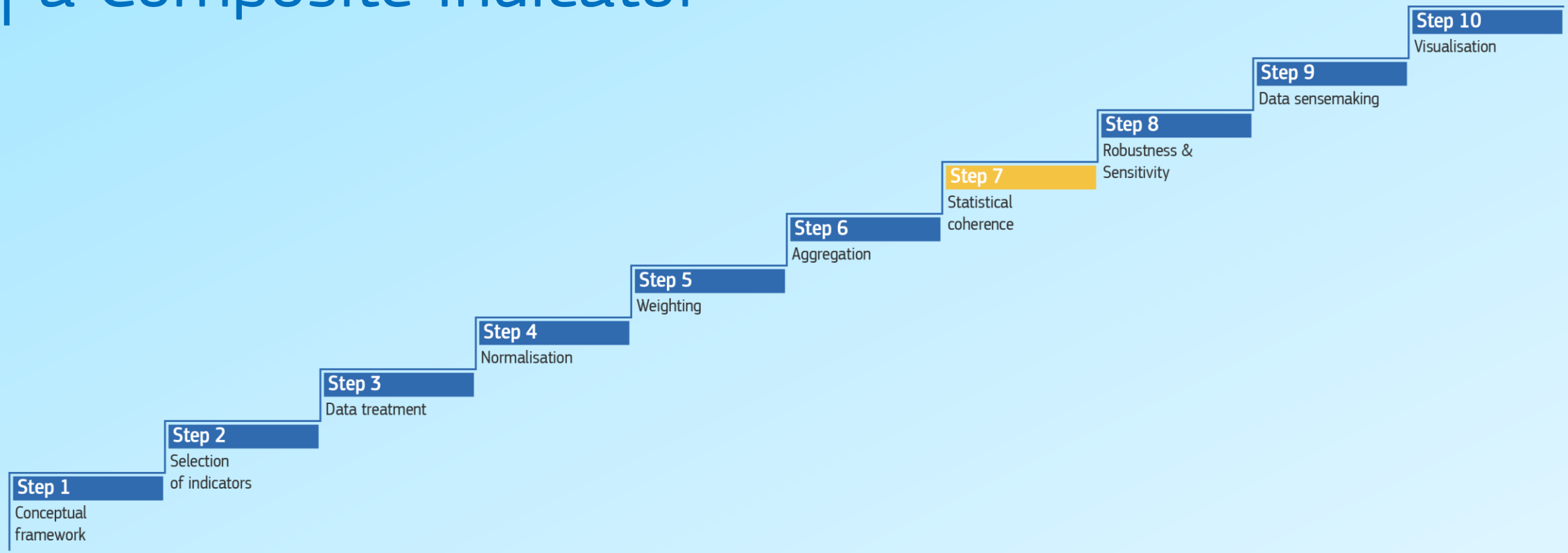
Step 7
Statistical coherence

18th JRC Annual training on Composite Indicators and Scoreboards

Eleni Papadimitriou

10 STEPS to build a Composite Indicator



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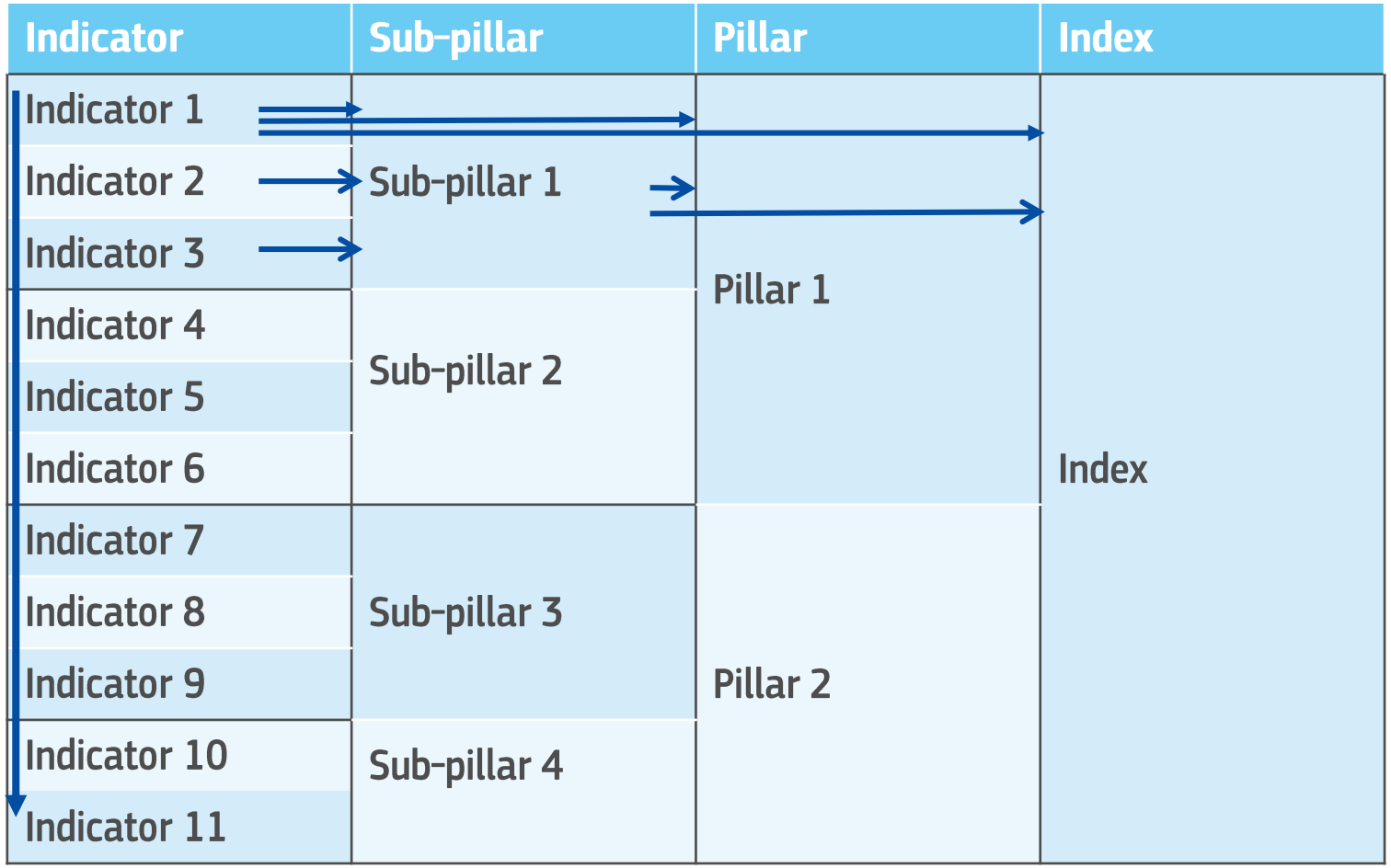
Outline

- Correlation analysis
- Principal component analysis
- Reliability analysis (Cronbach's Alfa)

Correlation analysis

Internal coherence of the framework



CORRELATION

$\rho(x,y)=Cov\ (x,y)/(\sigma x\ \sigma y)$

To which extend two variables are **linearly** related

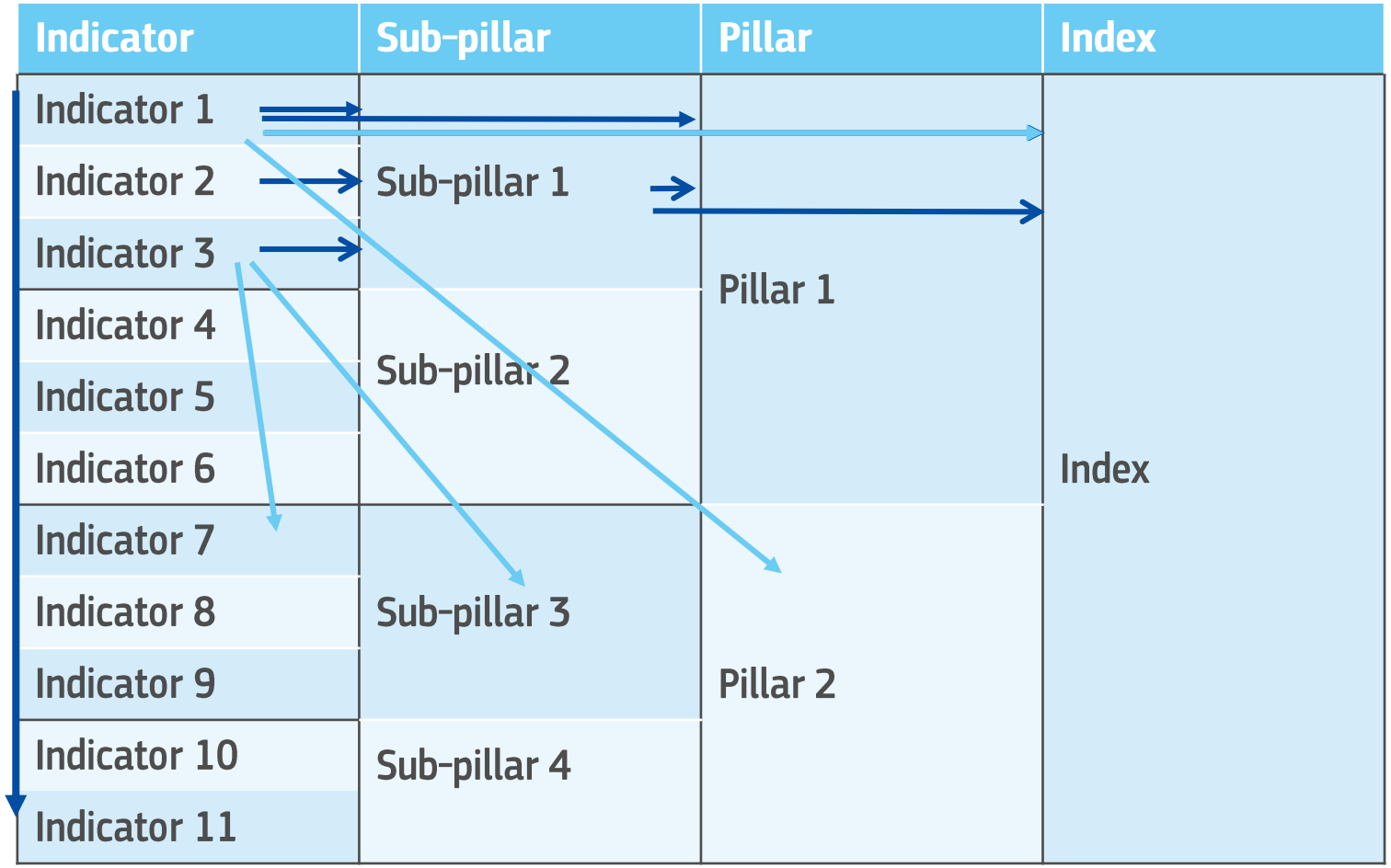
1 perfect positive linear correlation.

-1 perfect negative linear correlation.

0 no linear correlation.

- ρ^2 is the fraction of the variance in Y that is explained by X in a simple linear regression.

Which correlations do we observe?



Check whether indicators:

- Are negatively related corr < -0.3
- Are under-represented -0.3 < x < 0.3
- Dominate the framework

What responses we can find?

- Are indicators allocated in the **right dimension**?
- Are some indicators **over or under represented** in the aggregate **Index**?
- Up to what **level should we aggregate**?

An example – Gender Equality Index 2020

Indicator	Sub-domain	Domain	Index
Indicators 1 - 2	Participation	Work	Gender Equality Index
Indicators 3 – 5	Segregation and quality of work		
Indicators 6 – 7	Financial resources	Money	
Indicators 8 – 9	Economic situation		
Indicators 10 – 11	Attainment and participation	Knowledge	
Indicator 12	Segregation		
Indicators 13 – 14	Care activities	Time	
Indicators 15 – 16	Social activities		
Indicators 17 – 19	Political	Power	
Indicators 20 – 21	Economic		
Indicators 22 – 24	Social		
Indicators 25 – 27	Status	Health	
Indicators 28 – 29	Behavior		
Indicators 30 - 31	Access		

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Correlation at indicator level

	Indicator	Ministers	Parliament	Regional	Boards	Banks	Research	Media	Sport
Political power	Ministers	1.00	0.78	0.79	0.68	0.60	0.57	0.45	0.68
	Parliament	0.78	1.00	0.84	0.74	0.49	0.54	0.29	0.66
	Regional	0.79	0.84	1.00	0.69	0.57	0.57	0.38	0.70
Economic power	Boards	0.68	0.74	0.69	1.00	0.42	0.40	0.27	0.62
	Banks	0.60	0.49	0.57	0.42	1.00	0.52	0.44	0.62
Social power	Research	0.57	0.54	0.57	0.40	0.52	1.00	0.43	0.45
	Media	0.45	0.29	0.38	0.27	0.44	0.43	1.00	0.49
	Sports	0.68	0.66	0.70	0.62	0.62	0.45	0.49	1.00

Source: JRC Statistical Audit of the 2020 Gender Equality Index, Papadimitriou, E.; Norlén, H.; Del Sorbo, M.

Correlation at indicator level

	Indicator	Ind25	Ind26	Ind27	Ind28	Ind29	Ind30	IndX
Sub-domain 12	Indicator 25	1.00	0.77	0.65	0.31	0.31	0.35	-0.40
	Indicator 26	0.77	1.00	0.55	0.54	0.59	0.43	-0.29
	Indicator 27	0.65	0.55	1.00	0.43	0.08	0.45	-0.32
Sub-domain 13	Indicator 28	0.31	0.54	0.43	1.00	0.22	0.41	-0.36
	Indicator 29	0.31	0.59	0.08	0.22	1.00	0.12	0.26
Sub-domain 14	Indicator 30	0.35	0.43	0.45	0.41	0.12	1.00	-0.18
	Indicator x	-0.40	-0.29	-0.32	-0.36	0.26	-0.18	1.00

Cross -Correlations

-Are indicators allocated in the **right dimension?**
-Are some indicators **over or under represented** in the aggregate score?

	SD1	SD2	SD3	SD4	SD5	SD6	SD7	SD8	SD9	SD10	SD11	SD12	SD13	SD14	Work	Money	Knowledge	Time	Power	Health	INDEX
ind1	0.94	-0.06	-0.09	0.06	0.20	-0.20	0.45	0.12	0.15	0.07	0.31	-0.40	0.08	-0.21	0.46	-0.06	-0.01	0.25	0.18	-0.07	0.17
ind2	0.91	0.28	0.17	0.01	0.48	-0.24	0.66	0.46	0.54	0.32	0.47	-0.31	0.35	-0.26	0.72	0.16	0.12	0.57	0.48	0.16	0.49
ind3	-0.02	0.79	0.85	0.19	0.71	0.54	0.34	0.58	0.51	0.43	0.43	0.67	0.67	0.38	0.61	0.78	0.78	0.55	0.51	0.75	0.72
ind4	0.03	0.90	0.50	0.00	0.47	0.39	0.43	0.57	0.42	0.28	0.52	0.38	0.42	0.29	0.72	0.43	0.54	0.56	0.43	0.46	0.61
ind5	0.40	0.44	0.31	0.13	0.34	0.35	0.49	0.41	0.28	0.12	0.44	-0.02	0.21	0.13	0.57	0.29	0.44	0.48	0.28	0.15	0.44
ind6	-0.06	0.72	0.98	0.33	0.71	0.50	0.38	0.68	0.52	0.43	0.48	0.69	0.67	0.48	0.54	0.93	0.75	0.63	0.52	0.77	0.77
ind7	0.12	0.71	0.98	0.31	0.79	0.42	0.46	0.70	0.55	0.42	0.47	0.58	0.74	0.49	0.63	0.92	0.74	0.68	0.52	0.80	0.78
ind8	-0.11	0.13	0.41	0.89	0.31	0.30	-0.17	0.31	0.21	0.14	-0.10	0.46	0.46	0.44	0.04	0.63	0.39	0.16	0.11	0.54	0.26
ind9	0.09	0.09	0.29	0.99	0.26	0.20	-0.07	0.41	0.12	-0.06	-0.14	0.28	0.46	0.31	0.12	0.57	0.29	0.27	-0.02	0.49	0.20
ind10	0.23	0.61	0.67	0.17	0.91	0.20	0.40	0.52	0.37	0.24	0.41	0.47	0.51	0.26	0.61	0.62	0.67	0.53	0.37	0.56	0.60
ind11	0.42	0.59	0.72	0.35	0.89	0.26	0.63	0.79	0.64	0.42	0.47	0.28	0.69	0.16	0.69	0.73	0.69	0.81	0.56	0.64	0.78
ind12	-0.23	0.55	0.47	0.23	0.25	1.00	0.13	0.36	0.31	0.32	0.25	0.59	0.38	0.43	0.30	0.47	0.83	0.31	0.33	0.49	0.50
ind13	0.54	0.23	0.27	-0.13	0.31	-0.05	0.73	0.47	0.29	0.38	0.44	-0.16	0.32	-0.22	0.48	0.18	0.15	0.60	0.42	0.19	0.45
ind14	0.51	0.57	0.43	-0.06	0.58	0.19	0.94	0.64	0.48	0.39	0.57	0.09	0.32	-0.20	0.73	0.35	0.47	0.81	0.51	0.24	0.66
ind15	0.26	0.73	0.73	0.29	0.77	0.42	0.68	0.95	0.69	0.53	0.56	0.41	0.65	0.24	0.71	0.71	0.74	0.94	0.65	0.65	0.87
ind16	0.31	0.54	0.57	0.49	0.53	0.22	0.54	0.91	0.47	0.33	0.43	0.37	0.59	0.21	0.59	0.65	0.46	0.84	0.45	0.58	0.67

Source: JRC Statistical Audit of the 2020 Gender Equality Index, Papadimitriou, E.; Norlén, H.; Del Sorbo, M.

Upper levels: Sub-domain, Domain and Index

	SD1	SD2	SD3	SD4	SD5	SD6	SD7	SD8	SD9	SD10	SD11	SD12	SD13	SD14	D1	D2	D3	D4	D5	D6	GEI
SD1	1.00	0.10	0.03	0.04	0.35	-0.23	0.59	0.30	0.35	0.20	0.41	-0.39	0.22	-0.25	0.63	0.04	0.05	0.43	0.34	0.04	0.34
SD2	0.10	1.00	0.74	0.11	0.67	0.55	0.52	0.69	0.54	0.38	0.61	0.52	0.59	0.37	0.84	0.66	0.76	0.69	0.55	0.64	0.79
SD3	0.03	0.74	1.00	0.33	0.77	0.47	0.43	0.71	0.55	0.44	0.49	0.65	0.72	0.50	0.60	0.95	0.76	0.67	0.53	0.80	0.79
SD4	0.04	0.11	0.33	1.00	0.28	0.23	-0.10	0.40	0.15	-0.01	-0.13	0.34	0.48	0.35	0.10	0.61	0.32	0.25	0.02	0.52	0.22
SD5	0.35	0.67	0.77	0.28	1.00	0.25	0.57	0.72	0.55	0.36	0.49	0.42	0.66	0.24	0.72	0.74	0.75	0.73	0.50	0.67	0.76
SD6	-0.23	0.55	0.47	0.23	0.25	1.00	0.13	0.36	0.31	0.32	0.25	0.59	0.38	0.43	0.30	0.47	0.83	0.31	0.33	0.49	0.50
SD7	0.59	0.52	0.43	-0.10	0.57	0.13	1.00	0.66	0.48	0.44	0.60	0.01	0.37	-0.24	0.74	0.34	0.41	0.84	0.55	0.25	0.68
SD8	0.30	0.69	0.71	0.40	0.72	0.36	0.66	1.00	0.64	0.48	0.54	0.42	0.67	0.24	0.71	0.74	0.66	0.96	0.61	0.67	0.85
SD9	0.35	0.54	0.55	0.15	0.55	0.31	0.48	0.64	1.00	0.78	0.71	0.27	0.42	0.17	0.63	0.52	0.54	0.64	0.91	0.41	0.86
SD10	0.20	0.38	0.44	-0.01	0.36	0.32	0.44	0.48	0.78	1.00	0.70	0.31	0.33	0.10	0.42	0.38	0.43	0.51	0.93	0.35	0.77
SD11	0.41	0.61	0.49	-0.13	0.49	0.25	0.60	0.54	0.71	0.70	1.00	0.25	0.21	0.15	0.71	0.37	0.45	0.60	0.87	0.23	0.80
SD12	-0.39	0.52	0.65	0.34	0.42	0.59	0.01	0.42	0.27	0.31	0.25	1.00	0.45	0.56	0.19	0.66	0.65	0.31	0.31	0.67	0.48
SD13	0.22	0.59	0.72	0.48	0.66	0.38	0.37	0.67	0.42	0.33	0.21	0.45	1.00	0.32	0.59	0.77	0.65	0.63	0.37	0.96	0.64
SD14	-0.25	0.37	0.50	0.35	0.24	0.43	-0.24	0.24	0.17	0.10	0.15	0.56	0.32	1.00	0.16	0.53	0.44	0.09	0.15	0.51	0.28
D1															1.00	0.55	0.63	0.78	0.63	0.52	0.81
D2															0.55	1.00	0.75	0.66	0.47	0.85	0.75
D3															0.63	0.75	1.00	0.64	0.52	0.72	0.78
D4															0.78	0.66	0.64	1.00	0.64	0.58	0.86
D5															0.63	0.47	0.52	0.64	1.00	0.38	0.89
D6															0.52	0.85	0.72	0.58	0.38	1.00	0.66
GEI															0.81	0.75	0.78	0.86	0.89	0.66	1.00

Source: JRC Statistical Audit of the 2020 Gender Equality Index, Papadimitriou, E.; Norlén, H.; Del Sorbo, M.

Upper levels- another example

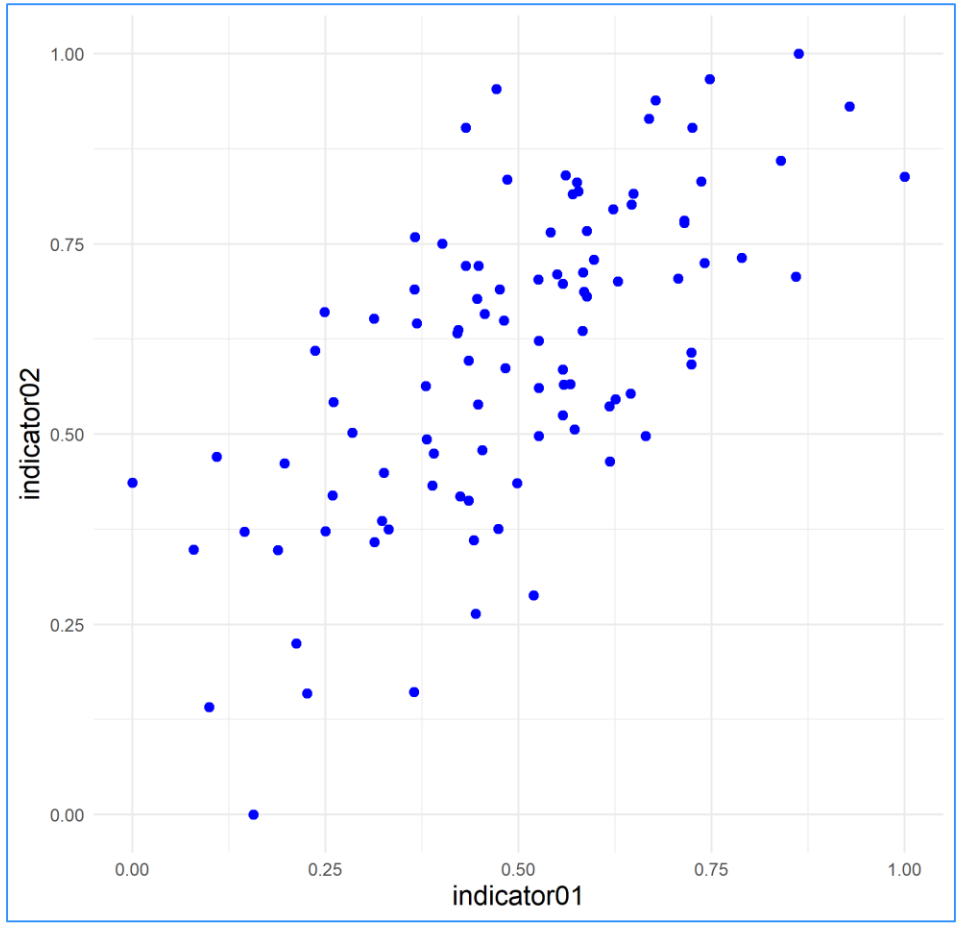
Dimension	1	2	3	4	5	6	7	8	9	10	11	12	13	14	Index
1	1.00														
2	0.49	1.00													
3	0.84	0.64	1.00												
4	0.77	0.61	0.84	1.00											
5	0.35	0.54	0.59	0.61	1.00										
6	0.69	0.66	0.81	0.73	0.68	1.00									
7	0.88	0.50	0.85	0.81	0.46	0.71	1.00								
8	0.50	0.60	0.68	0.62	0.59	0.63	0.51	1.00							
9	0.66	0.66	0.82	0.67	0.59	0.75	0.68	0.63	1.00						
10	0.36	0.29	0.36	0.17	0.01	0.21	0.19	0.24	0.38	1.00					
11	0.54	0.46	0.68	0.66	0.64	0.69	0.59	0.53	0.55	0.10	1.00				
12	-0.59	-0.53	-0.76	-0.59	-0.52	-0.67	-0.60	-0.46	-0.86	-0.32	-0.50	1.00			
13	-0.28	-0.17	-0.33	-0.32	-0.18	-0.19	-0.27	-0.10	-0.32	-0.05	-0.13	0.50	1.00		
14	-0.17	-0.01	-0.11	-0.10	0.09	-0.04	-0.13	0.06	-0.05	-0.19	-0.04	0.04	0.02	1.00	
Index	0.84	0.71	0.93	0.86	0.67	0.86	0.86	0.73	0.83	0.40	0.73	-0.68	-0.20	-0.01	1.00

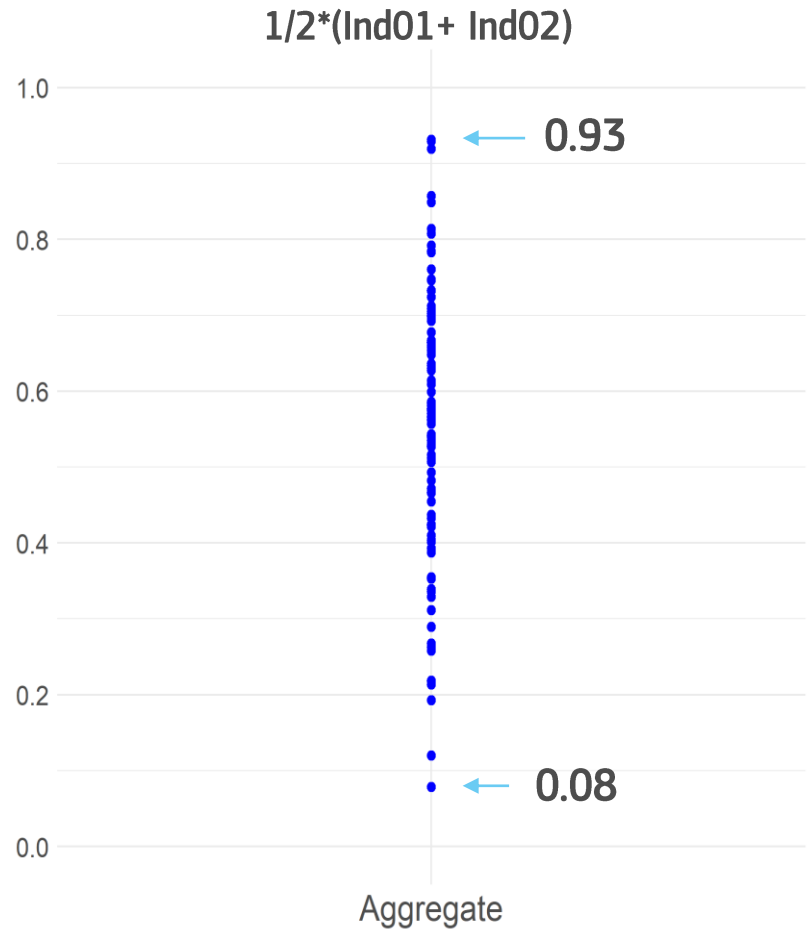
- Corr >0.7 the index captures at least 50% of the variation in the underlying dimension and vice versa (Dimensions 1-9 and 11).
- Very high correlation (0.93) may suggest that dimension 3 is driving the index or telling the entire story.
- Negative relationship with the index. Reflect on dims 12-13: keep them apart? Stop the aggregation at dim level?

Why correlations are important?

Indicator 1
Indicator 2

Correlation
0.663





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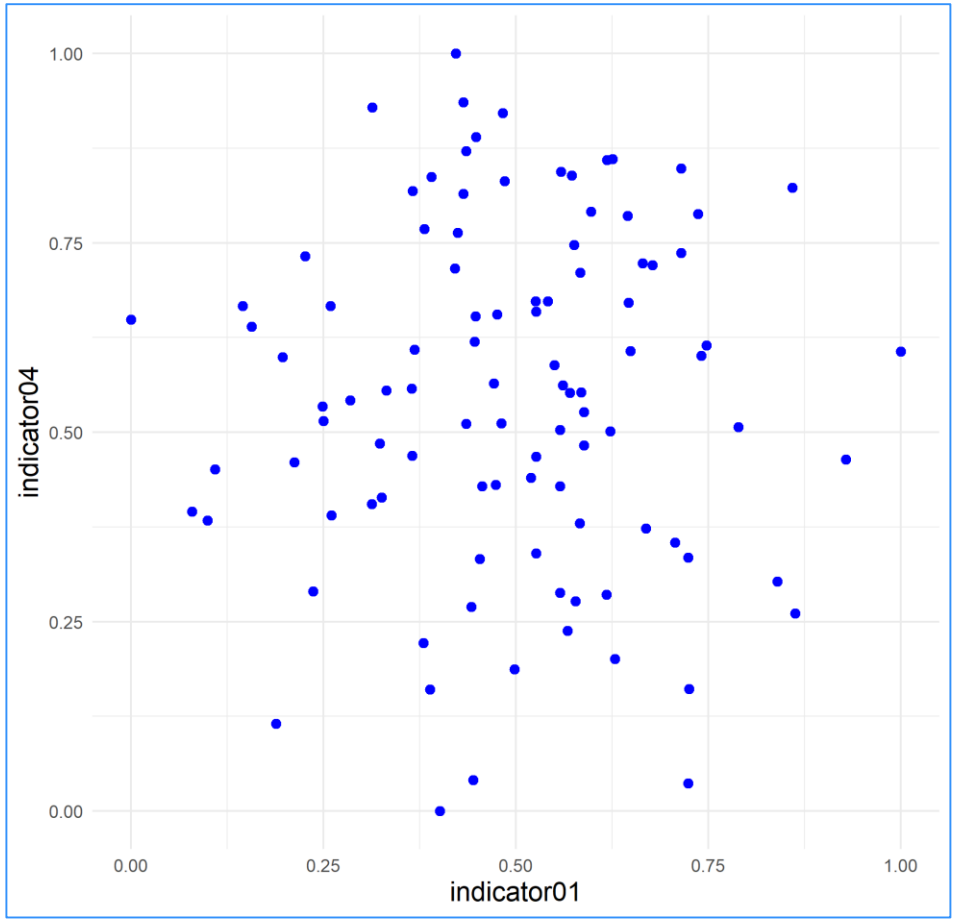
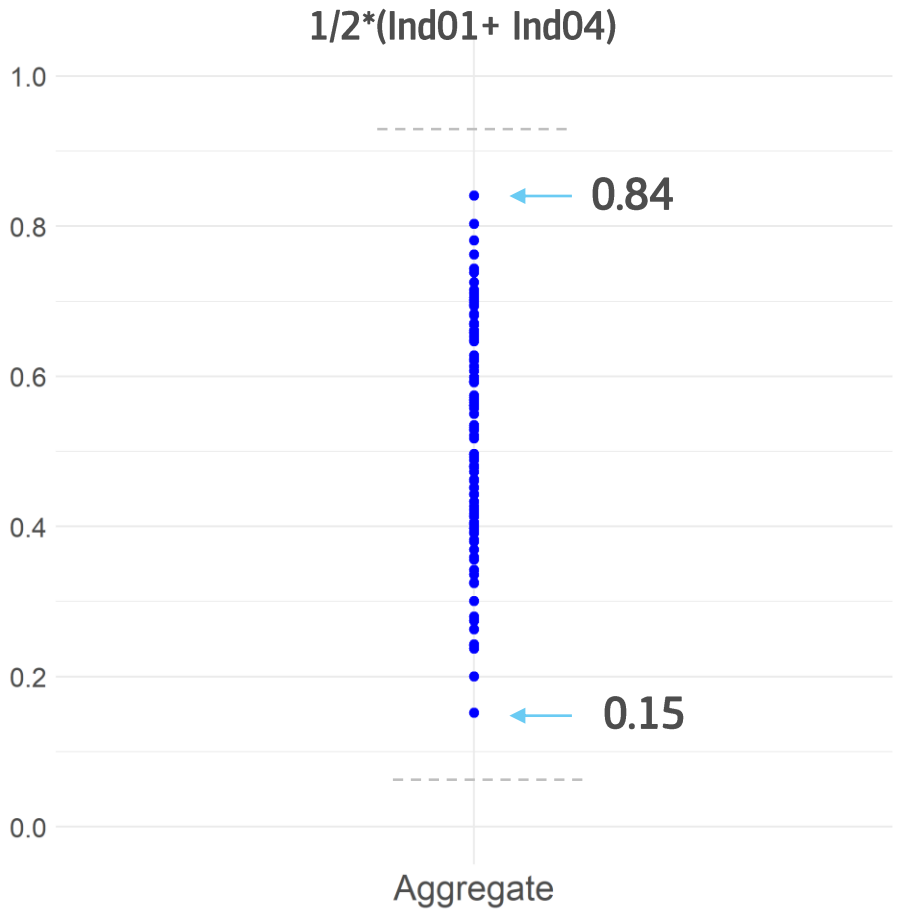
Why correlations are important?

Indicator 1
Indicator 4

Correlation
0.011





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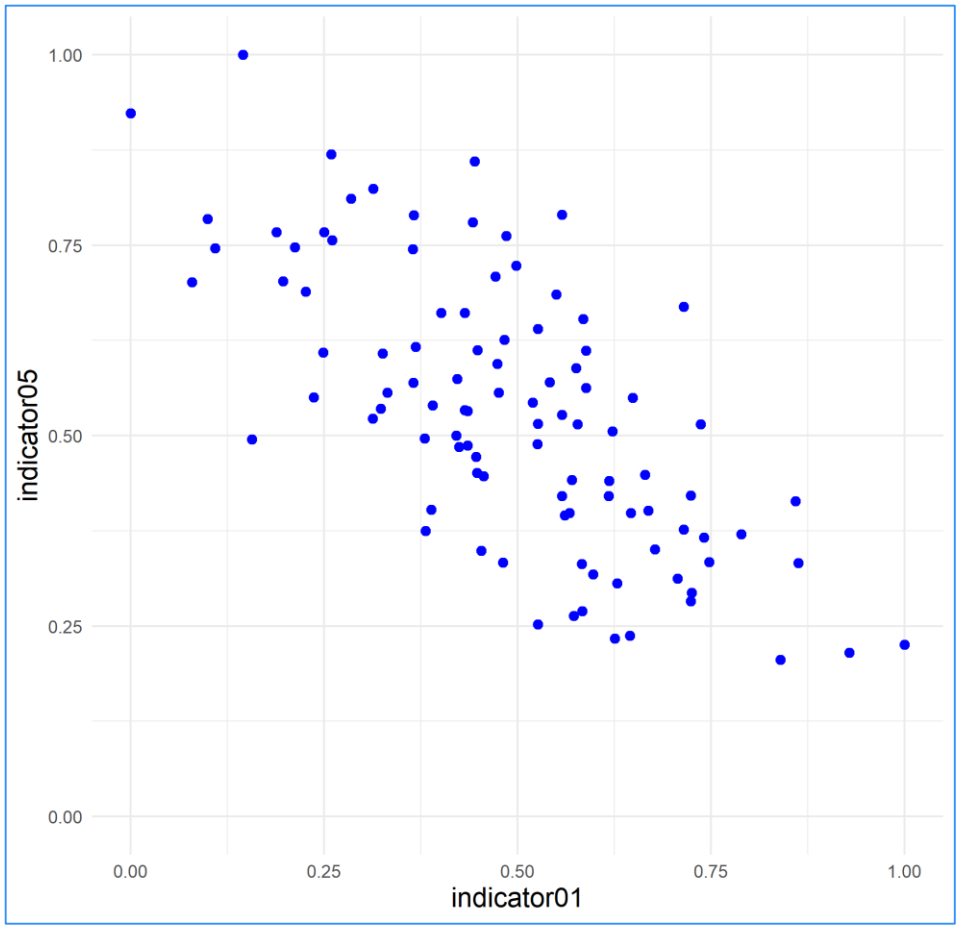
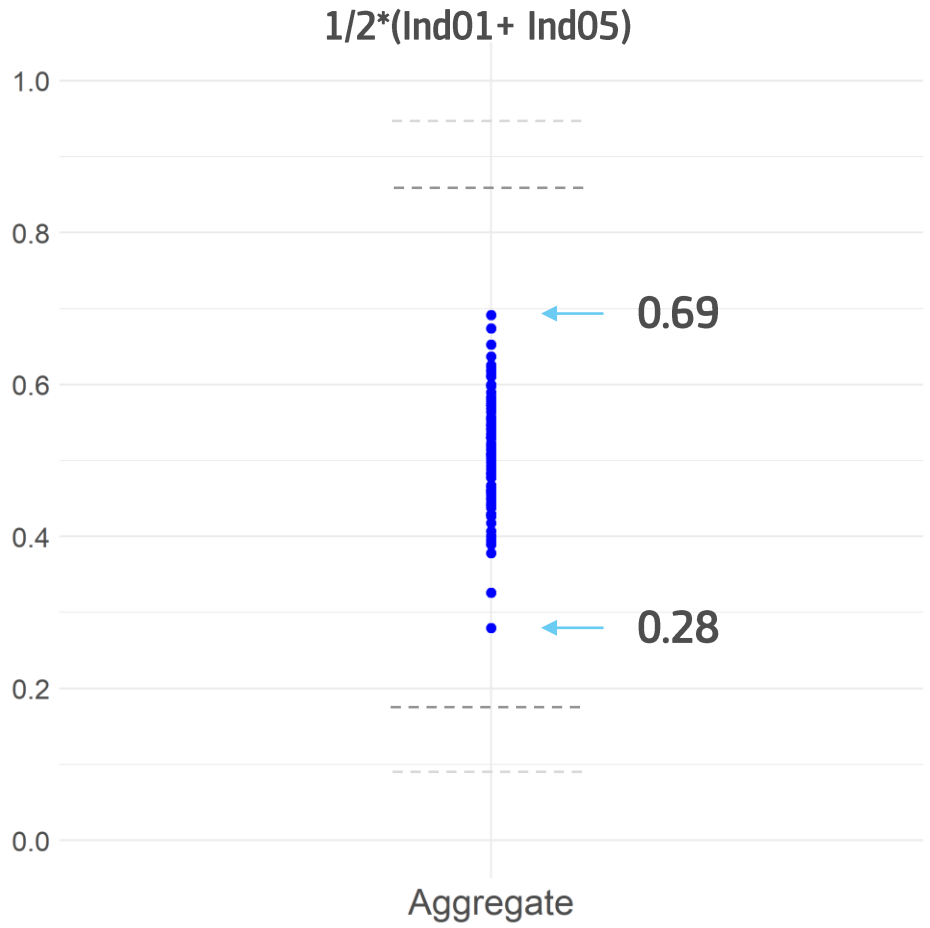
Why correlations are important?

Indicator 1
Indicator 5

Correlation
-0.681





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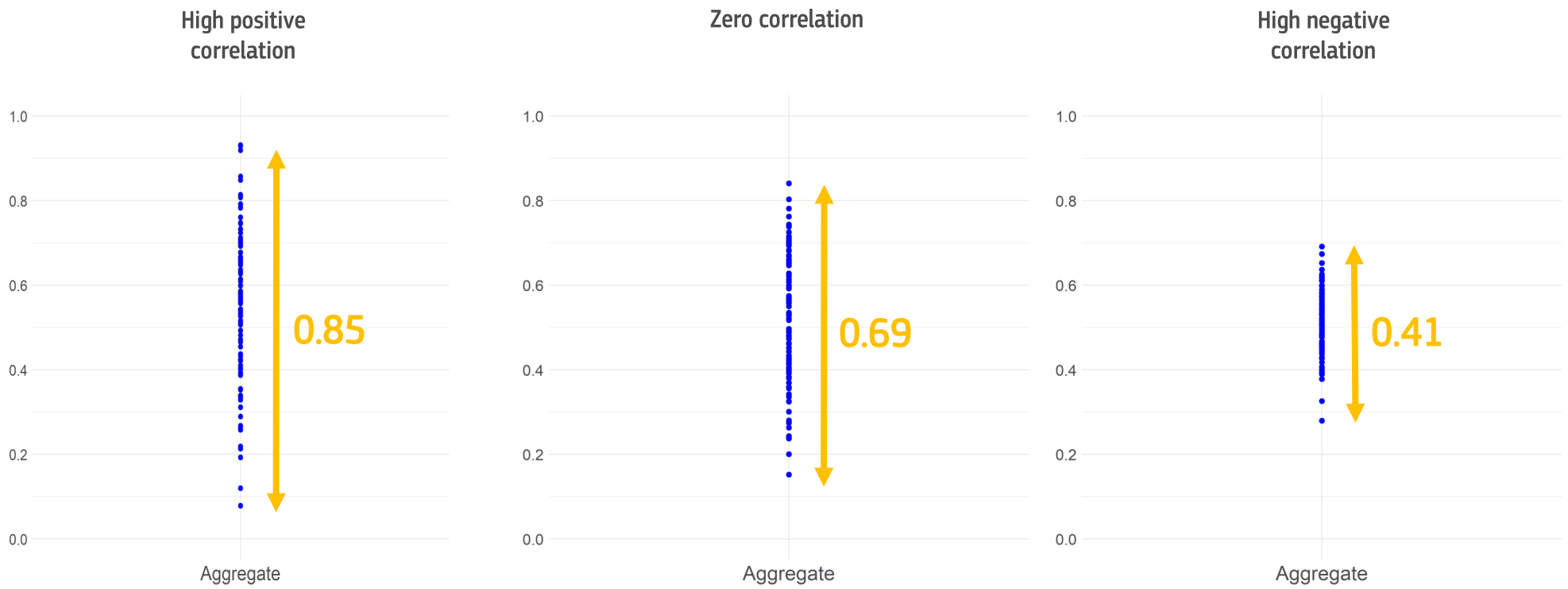
Why correlations are important?



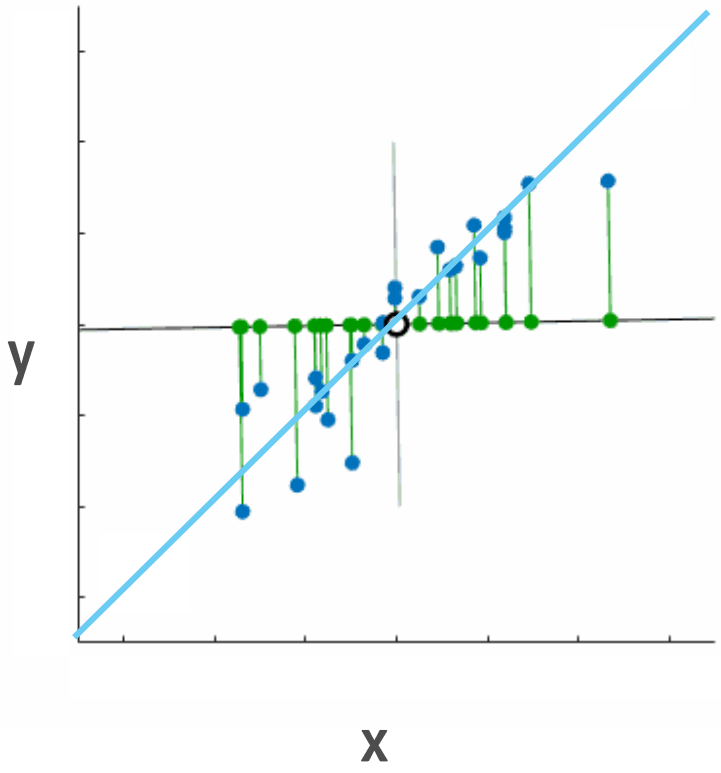
The range constitutes an implicit weight of the aggregates!

Take home message

- The analysis of correlation alone is not the final source of every decision.
- When an inconsistency is spotted, ask yourself why.
- Is never too late for a revision or an improvement of the theoretical framework.
- Acknowledge the inconsistencies that cannot be eliminated.
- Be transparent about your choices! Transparency is the main source of reliability of a Composite Indicator.

Principal Component Analysis

B. Principal Component Analysis (PCA)



We have data (more than one variable)

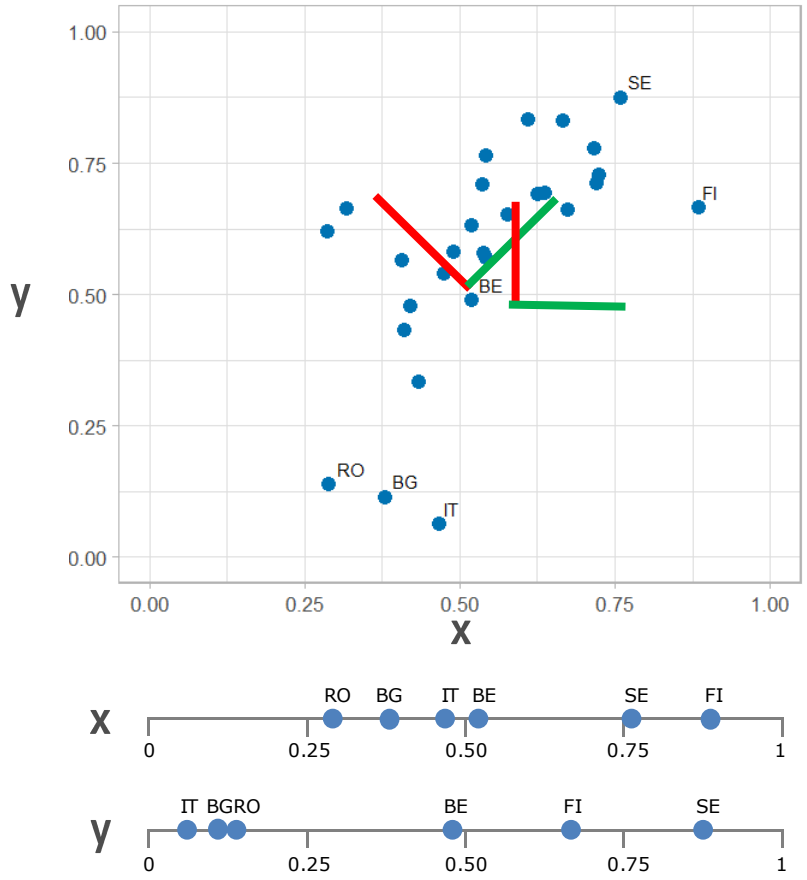
We want to describe the data with less variables!

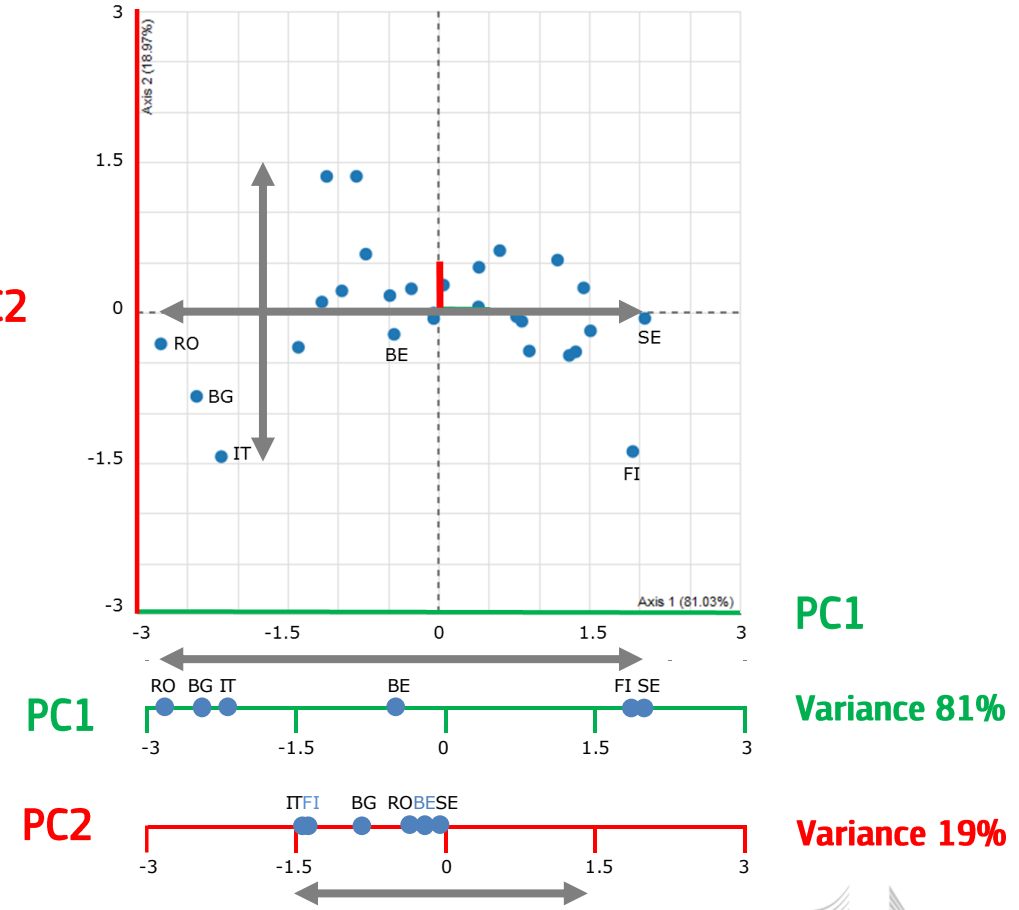
PCA will finds this "best" line:

1) Maximum variance
2) Minimum error

First Principal Component!

Example with two variables





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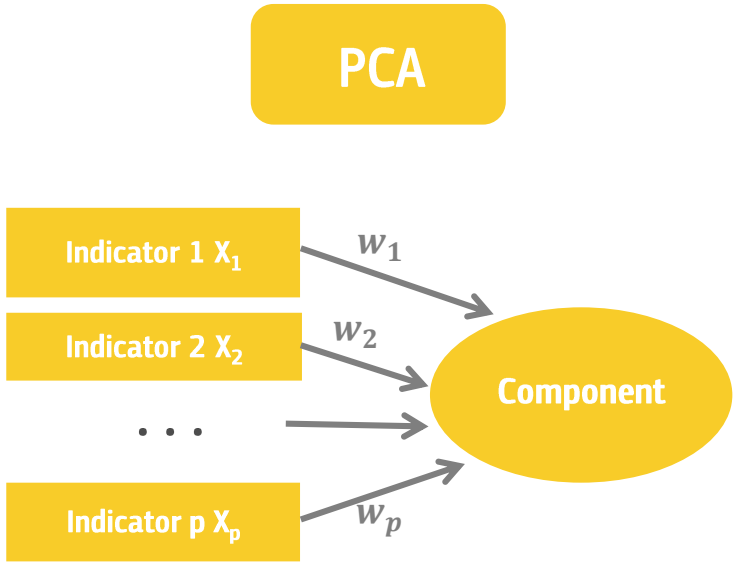
Principal Component Analysis



Observed indicators are reduced into components

Each principal component PC_i is a new variable computed as a linear combination of the original (standardized) variables

$$PC_1 = w_1x_1 + w_2x_2 + \cdots w_px_p$$

The principal components are orthogonal to each other

PCA summarizes information of all indicators and reduces it into a fewer number of components

The magic number.

How many components do we keep in PCA?

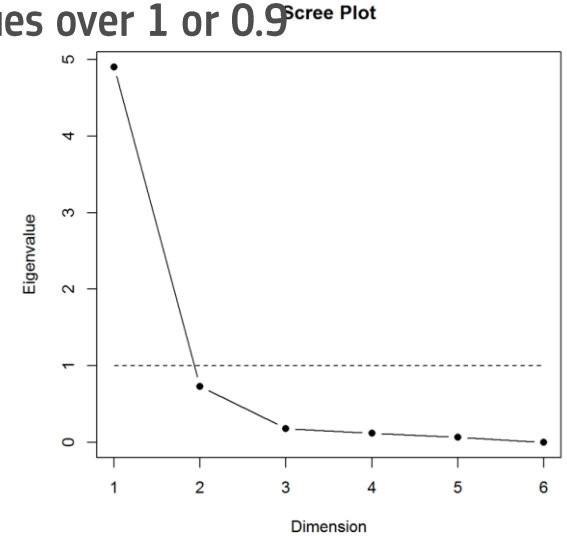
Several methods exist. The 3 most common are:

1) **Certain percentage of explained variance**
   e.g., >2/3, 75%, 80%,…

2) **Kaiser–Guttman 'Eigenvalues greater than one' criterion** (Guttman (1954), Kaiser (1960)). Select all components with eigenvalues over 1 or 0.9

3) **Cattell's scree test**

   (Cattell (1966)) "Above the elbow" approach.



How we use PCA

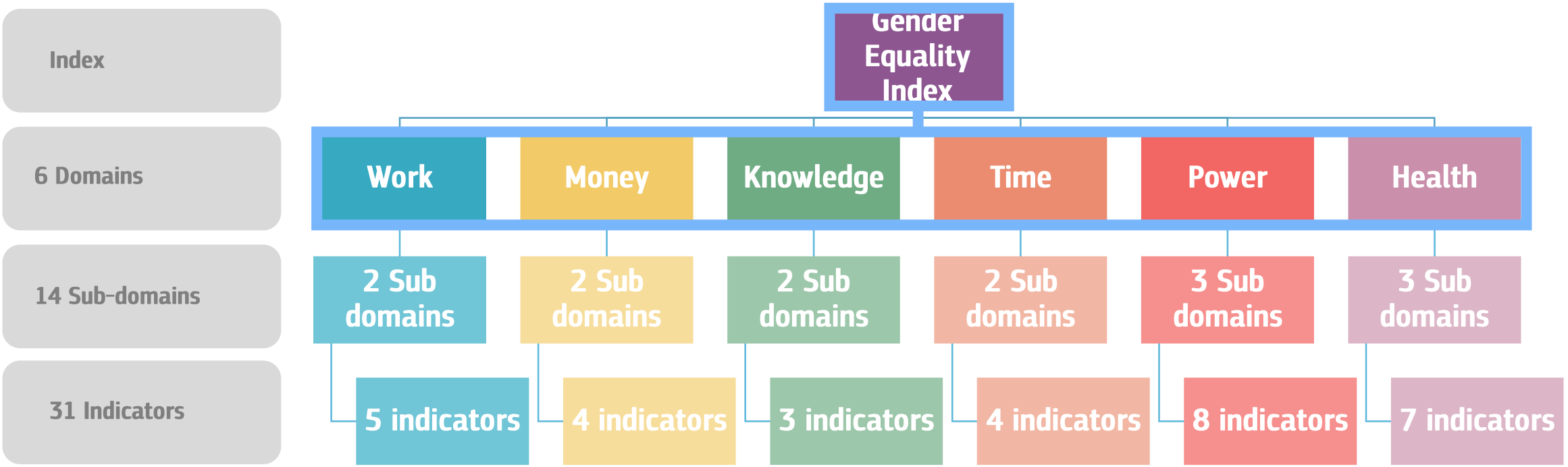
A **Composite Indicator** measures multifaceted phenomenon - combination of different aspects (**Sub-pillars/Pillars**).

Each aspect can be measured by a set of observable variables (**Indicators**).

PCA *is* used to verify the internal consistency, verify "**unidimensionality**" within:

1) each Sub-pillar (across Indicators)
2) each Pillar (across Sub-pillars)
3) the Composite Indicator (across Pillars)

Gender Equality Index



Gender Equality Index – PCA results

Check the correlation structure

Correlation matrix	Work	Money	Knowledge	Time	Power	Health
Work	1					
Money	0.55	1				
Knowledge	0.63	0.75	1			
Time	0.78	0.66	0.64	1		
Power	0.63	0.47	0.52	0.64	1	
Health	0.52	0.85	0.72	0.58	0.38	1

Pearson correlation coefficients

Gender Equality Index – PCA results

$4.11 = 0.82^2 + 0.87^2 + 0.86^2 + 0.87^2 + 0.72^2 + 0.82^2$

$(4.11/6)*100$

Total variance explained

Principal Component	Eigenvalue	Variance	Cumulative variance
PC1	**4.11**	**68.54**	**68.54**
PC2	0.86	14.28	82.82
PC3	0.39	6.58	89.40
PC4	0.29	4.91	94.31
PC5	0.21	3.48	97.79
PC6	0.13	2.21	100
Sum	6.00	100.00	

Pearson correlation coefficients between domains and principal components

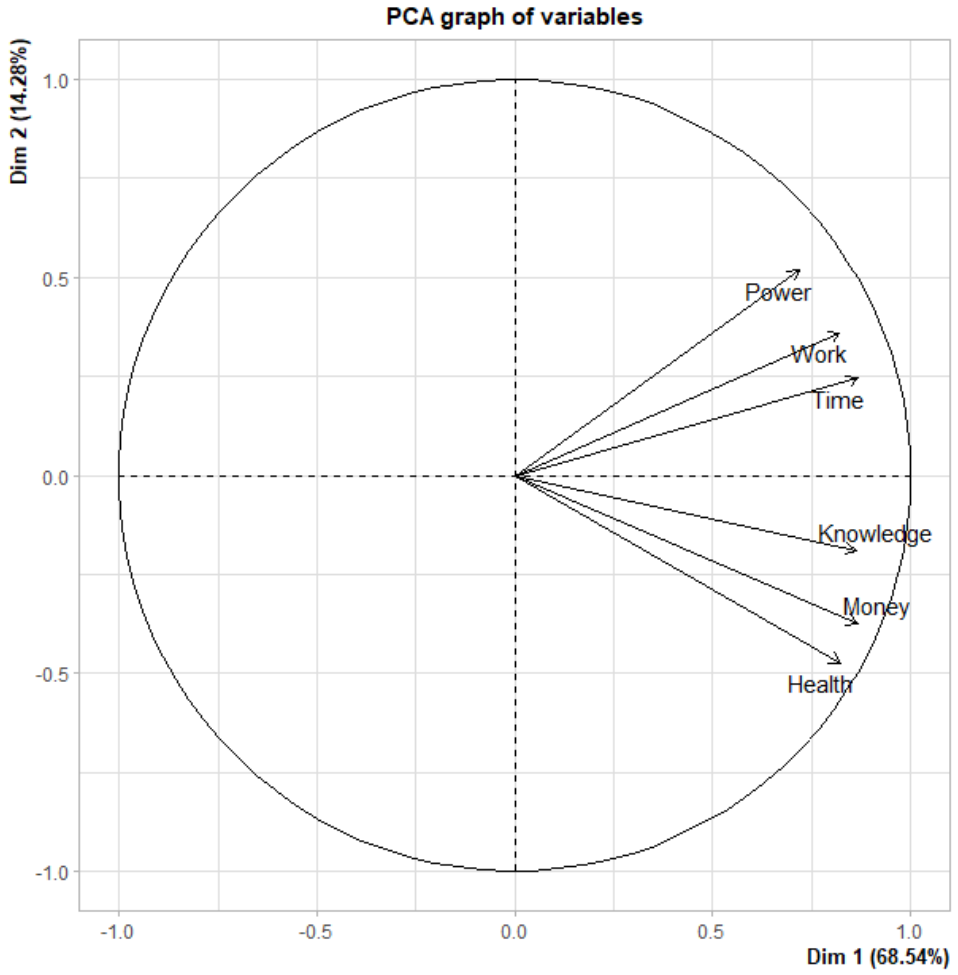
	PC1	PC2	PC3	...
Work	**0.82**	0.36	-0.34	
Money	**0.87**	-0.37	0.09	
Knowledge	**0.86**	-0.19	0.07	
Time	**0.87**	0.24	-0.23	
Power	**0.72**	0.52	0.46	
Health	**0.82**	-0.47	0.02	
Sum of Squares	4.11	0.86	0.39	

Stopping criterion: eigenvalue >1

One dimension verified!

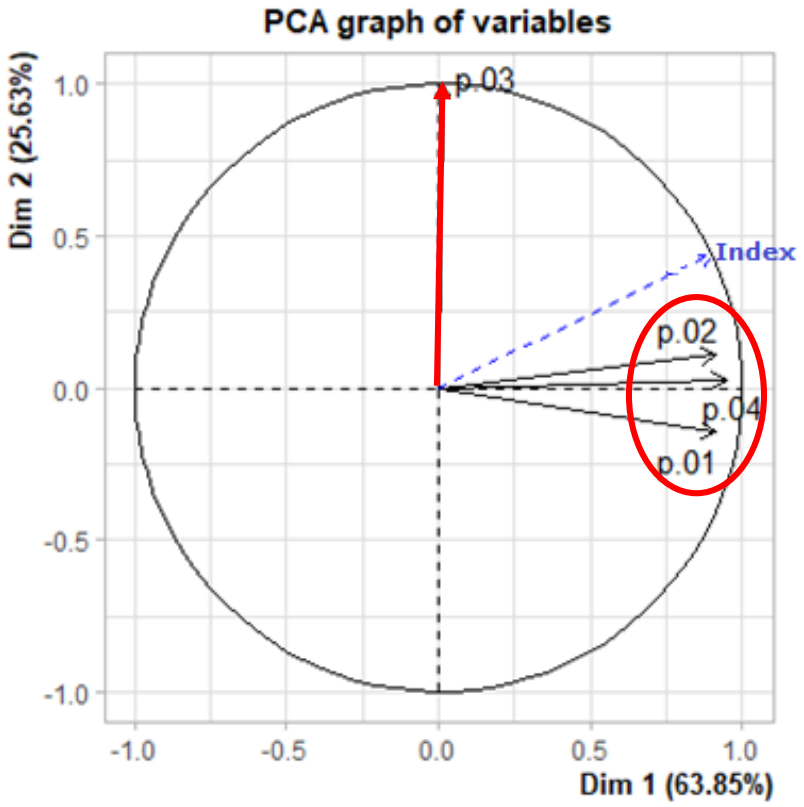
$$PC1 = 0.82 * Work + 0.87 * Money + 0.86 * Knowledge + 0.87 * Time + 0.72 * Power + 0.82 * Health$$

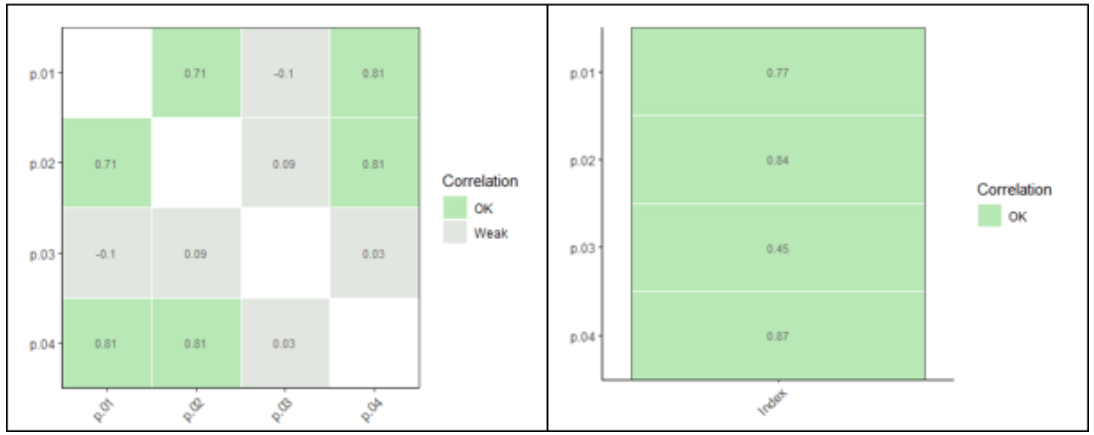
Gender Equality Index – PCA results



PCA results often summarized in a “Factor map”, plotted against the first two principal components

PCA - Example





Principal Component	Eigenvalue	Variance	Cumulative variance
PC1	2.55	63.85	63.85
PC2	1.03	25.63	89.48
PC3	0.26	6.57	96.05
PC4	0.16	3.95	100
Sum	4	100	

	PC1
Pillar 1	0.91
Pillar 2	0.91
Pillar 3	0.01
Pillar 4	0.95

Take home message

- Reflect on the result. Ask yourself why.
- Go back to the conceptual framework and the previous steps!
- Be transparent about your choices!

Reliability Analysis

C. Reliability Analysis

Cronbach's α is measure of internal consistency and reliability

α is defined as Ratio of the Total Covariance in a set of indicators to the Total Variance in the set.

$$\alpha = \frac{k}{k-1}\left(1 - \frac{\sum V_i}{V_t}\right)$$, k number of indicators

Caution! α increases when the number of indicators increases.

Cronbach's alpha	Internal consistency
0.9 ≤ α	Excellent
0.8 ≤ α < 0.9	Good
0.7 ≤ α < 0.8	Acceptable
0.6 ≤ α < 0.7	Questionable
0.5 ≤ α < 0.6	Poor
α < 0.5	Unacceptable

Thank you

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European Commission

Thank you

European Commission