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Step 6
Aggregation

18th JRC Annual training on Composite Indicators and Scoreboards

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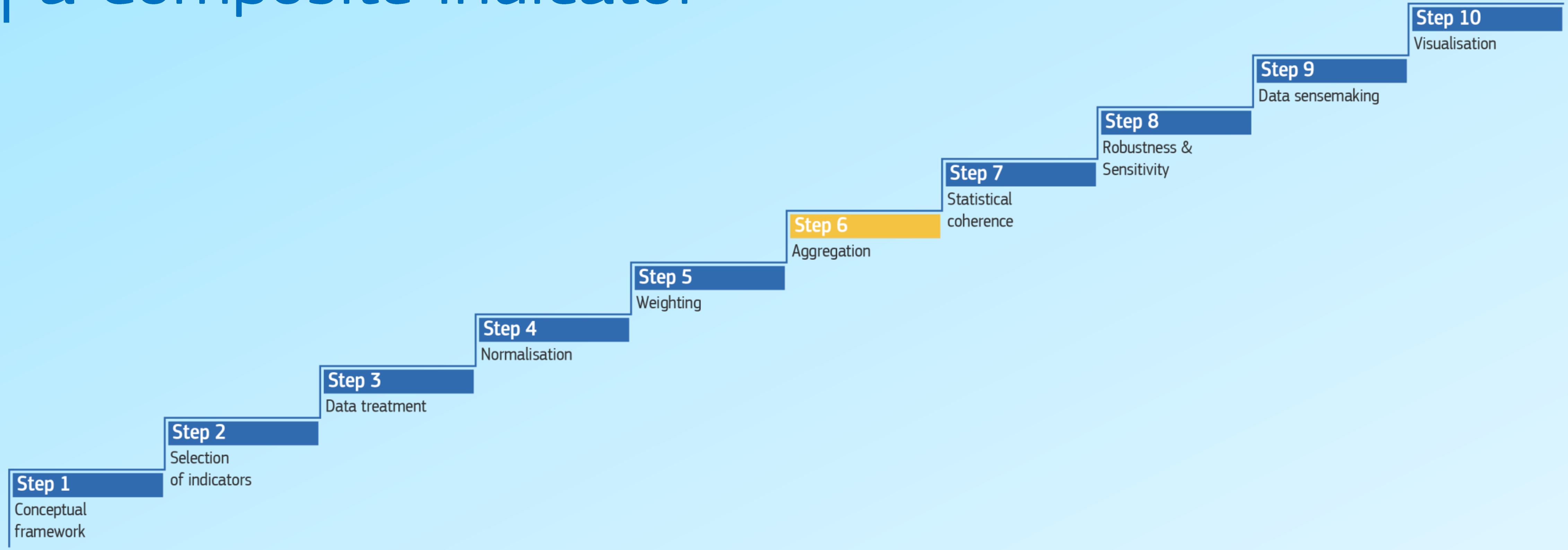
Joint Research Centre

10 STEPS to build a Composite Indicator



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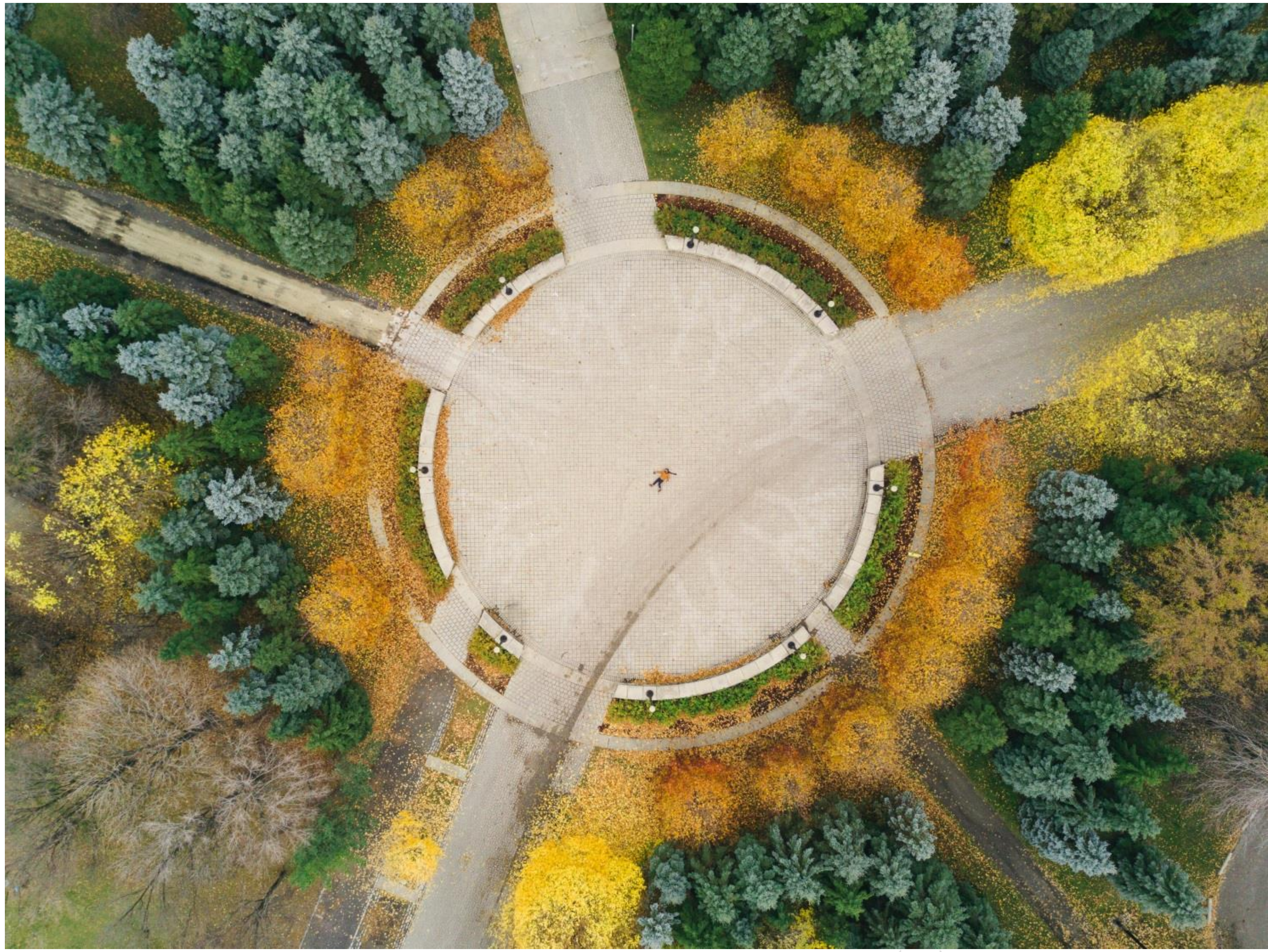
Aggregation methods

- Based on Average Values
- Based on Ranks
- Based on Pairwise comparison



Arithmetic mean

The arithmetic mean of a list of n real numbers equals:

$$\frac{1}{n}\sum_{i=1}^{n} x_i = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

This is the simplest, most obvious and most widespread aggregation method

Perfect (and constant) substitutability – underperformance in one component can be perfectly compensated by equivalent overperformance in another

Geometric mean

The ***geometric mean*** of a list of *n* *positive* real numbers equals:

$$\sqrt[n]{\prod_{i=1}^{n} x_i} = \sqrt[n]{x_1 \times x_2 \times \cdots \times x_n}$$

The first “less-compensatory” option

Partial substitutability – *unbalanced performance is always penalised* by the aggregation formula when compared to arithmetic aggregation

The formula is the decision approach

Safe: 1
Yummy: 10

A.M.: **5.5**
G.M.: 3.2

Safe: 5
Yummy: 5

A.M.: 5.0
G.M.: **5.0**

Safe: 9
Yummy: 2

A.M.: **5.5**
G.M.: 4.2

Geometric vs. Arithmetic mean: Implications

	Sufficient Food	Sufficient to Drink	Safe Sanitation	Basic Needs (arithmetic)	Country *i's* improvement	Basic Needs (geometric)	Country *i's* improvement
Country *i* (t)	10.0	8.6	1.4	6.7		4.9	

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The two averages

Common features:

- ***Not robust to Outliers***
- ***Normalisation***
- ***Quantitative information needed and kept***
- ***"Good enough" correlation structure is necessary***
- ***Weights*** are ***interpreted as trade-offs***

Differences:

- ***Perfect substitutability*** vs. ***partial substitutability***
- ***Arithmetic mean*** is always ***greater than or equal to*** the equivalent ***geometric mean***

Aggregation with an average

Before proceeding for a mean, check all the previous steps (normalisation, outliers, missing data, weights)

Remember: You need <u>*quantitative*</u> indicators

Qualitative and quantitative together

Multi-criteria performance matrix

	Criterion 1 (/20)	Criterion 2 (rating)	Criterion 3 (qual.)	Criterion 4 (Y/N)
Alternative 1	20	135	G	Yes
Alternative 2	9	156	B	Yes
Alternative 3	15	129	VG	No
Alternative 4	9	146	VB	No
Alternative 5	7	121	G	Yes
…	…	…	…	…

We need some method to compare, rank or evaluate the alternatives

Methods based on ranks

Every criterion represents a voter, a point of view,
and determines a complete ranking

	Criterion 1 (/20)	Criterion 2 (rating)	Criterion 3 (qual.)	Rank 1	Rank 2	Rank 3
Alternative 1	20	135	Good	1	3	2.5
Alternative 2	9	156	Bad	3.5	1	4
Alternative 3	15	129	Very Good	2	4	1
Alternative 4	9	146	Very Bad	3.5	2	5
Alternative 5	7	121	Good	5	5	2.5

Median rank

3 units: **A, B, C**

11 indicators/criteria

5 indicators	**4** indicators	**2** indicators
A	C	C
C	A	B
B	B	A

Ordered ranks
A: 11111**2**22233
B:22333**3**33333
C:11111**1**22222

Ranking
C
A
B

Weighted median rank

3 units: **A, B, C**

6 indicators/ criteria **<u>with (unequal) weights</u>**

Rank	Ind. 1	Ind. 2	Ind. 3	Ind. 4	Ind. 5	Ind. 6
	0.05	**0.25**	**0.30**	**0.10**	**0.05**	**0.25**
1	A	C	C	B	A	B
2	C	A	B	A	B	C
3	B	B	A	C	C	A

	Cumulative weight			
Unit	**0.10**	**0.25**	**0.50**	**0.75**
A	1	2	**3**	3
B	1	1	**2**	3
C	1	1	**1**	2

Methods based on pairwise comparisons

	Criterion 1 (/20)	Criterion 2 (rating)	Criterion 3 (qual.)	Criterion 4 (Y/N)	…
Alternative 1	20	135	G	Yes	…
Alternative 2	9	156	B	Yes	…
Alternative 3	15	129	VG	No	…
Alternative 4	9	146	VB	No	…
Alternative 5	7	121	G	Yes	…
…	…	…	…	…	…

Compare alternatives using the original values, all criteria simultaneously

Example: Alternative 1 is better than Alternative 5

Outranking Matrix – Concordance value

Step 1 – Raw data, Weights & Orientation

	Index			
	Ind 01	Ind 02	Ind 03	Ind04
Orientation	-1	-1	-1	-1
Weights	0.25	0.25	0.25	0.25
MT	74.9	Good	205.0	14.7
NL	75.8	Acceptable	47.9	11.6
PL	58.9	Excellent	87.9	5.8
PT	124.3	Very good	445.5	18.8
RO	38.5	Very Poor	207.6	7.3
SE	39.0	None	17.7	7.0
SI	66.5	Outstanding	350.9	5.6
SK	56.7	Poor	87.8	9.2

Outranking Matrix – Concordance value

	Index			
	Ind 01	Ind 02	Ind 03	Ind04
Orientation	-1	-1	-1	-1
Weights	0.25	0.25	0.25	0.25
MT	74.9	Good	205.0	14.7
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PT	124.3	Very good	445.5	18.8
RO	38.5	Very Poor	207.6	7.3
SE	39.0	None	17.7	7.0
SI	66.5	Outstanding	350.9	5.6
SK	56.7	Poor	87.8	9.2

Step 1 – Raw data & Weights & Orientation

Step 2 – Concordance value

For *n* countries, there are *n (n-1)* pairwise comparisons to be made

Example:
MT versus NL = 0.25
NL versus MT = 0.75

Sum = 1.00

Outranking Matrix – Concordance value

	Index			
	Ind 01	Ind 02	Ind 03	Ind04
Orientation	-1	-1	-1	-1
Weights	0.25	0.25	0.25	0.25
MT	74.9	Good	205.0	14.7
NL	75.8	Acceptable	47.9	11.6
PL	58.9	Excellent	87.9	5.8
PT	124.3	Very good	445.5	18.8
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SK	56.7	Poor	87.8	9.2

Step 1 – Raw data & Weights & Orientation

Step 2 – Concordance value

For *n* countries, there are ***n (n-1)* pairwise comparisons** to be made

Example:
MT versus PT = 1.00
PT versus MT = 0.00

Sum = 1.00 → **Robust pair**

Outranking Matrix – Construction

Step 1 – Raw data & Weights & Orientation

Step 2 – Concordance value

Step 3 – **Outranking matrix**

	MT	NL	PL	PT	RO	SE	SI	SK
MT	0.00	0.25	0.25	1.00	0.25	0.00	0.50	0.00
NL	0.75	0.00	0.50	1.00	0.25	0.00	0.50	0.25
PL	0.75	0.50	0.00	0.75	0.50	0.25	0.75	0.50
PT	0.00	0.00	0.25	0.00	0.00	0.00	0.25	0.00
RO	0.75	0.75	0.50	1.00	0.00	0.25	0.75	0.75
SE	1.00	1.00	0.75	1.00	0.75	0.00	0.75	1.00
SI	0.50	0.50	0.25	0.75	0.25	0.25	0.00	0.25
SK	1.00	0.75	0.50	1.00	0.25	0.00	0.75	0.00

All concordance values are entered in the outranking matrix.
(entries above and below the diagonal sum up to 1.0)

MT versus NL = 0.25
NL versus MT = 0.75

MT versus PT = 1.00
PT versus MT = 0.00

The Copeland Score

Victories (+1) minus ***Defeats (-1)*** (***ties don't count***)

Outranking matrix

	MT	NL	PL	PT	RO	SE	SI	SK
MT	0.00	0.25	0.25	1.00	0.25	0.00	0.50	0.00
NL	0.75	0.00	0.50	1.00	0.25	0.00	0.50	0.25
PL	0.75	0.50	0.00	0.75	0.50	0.25	0.75	0.50
PT	0.00	0.00	0.25	0.00	0.00	0.00	0.25	0.00
RO	0.75	0.75	0.50	1.00	0.00	0.25	0.75	0.75
SE	1.00	1.00	0.75	1.00	0.75	0.00	0.75	1.00
SI	0.50	0.50	0.25	0.75	0.25	0.25	0.00	0.25
SK	1.00	0.75	0.50	1.00	0.25	0.00	0.75	0.00

	Wins	Defeats	Total	Rank
SE	7	0	7	1
RO				
SK				
PL				
NL				
SI				
MT				
PT				

The Copeland Score

Outranking matrix

	MT	NL	PL	PT	RO	SE	SI	SK
MT	0.00	0.25	0.25	1.00	0.25	0.00	0.50	0.00
NL	0.75	0.00	0.50	1.00	0.25	0.00	0.50	0.25
PL	0.75	0.50	0.00	0.75	0.50	0.25	0.75	0.50
PT	0.00	0.00	0.25	0.00	0.00	0.00	0.25	0.00
RO	0.75	0.75	0.50	1.00	0.00	0.25	0.75	0.75
SE	1.00	1.00	0.75	1.00	0.75	0.00	0.75	1.00
SI	0.50	0.50	0.25	0.75	0.25	0.25	0.00	0.25
SK	1.00	0.75	0.50	1.00	0.25	0.00	0.75	0.00

	Wins	Defeats	Total	Rank
SE	7	0	7	1
RO	5	1	4	2
SK	4	2	2	3
PL	3	1	2	4
NL	2	3	-1	5
SI	1	4	-3	6
MT	1	5	-4	7
PT	0	6	-6	8

This method is in the COIN tool and the COINr package

Summary of methods on Pairwise Comparison

- Fully ***non-compensatory*** approach;
- ***only weights and orientation*** are ***required*** to obtain the ranking of alternatives;
- ***weights*** represent exactly ***the importance*** *of the indicator*;
- no impact of outliers;
- no need for data normalisation
- no need for ”good” correlation structure;
- can be used both with continuous and categorical variables;
- computationally more demanding than standard averages;
- poor with small number of units;
- software available for Copeland (send an email to JRC-COIN): Excel, R, Matlab

Sources: Athanasoglou (2015) , Tarjan (1972), Van Zuylen, and Williamson (2009), Munda and Nardo (2009)

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Thank you

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Suggested readings

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