# Step 6
# Aggregation methods

19th JRC Annual training on Composite Indicators and Scoreboards

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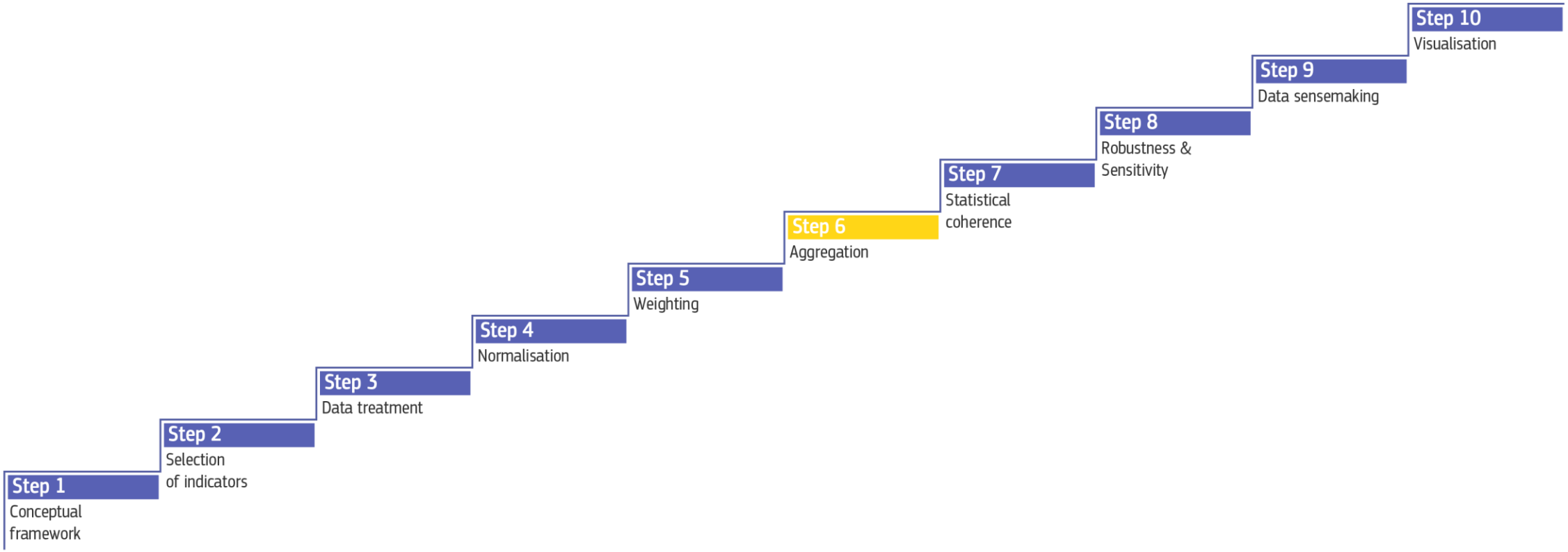
European Commission

# 10 STEPS to build a Composite Indicator

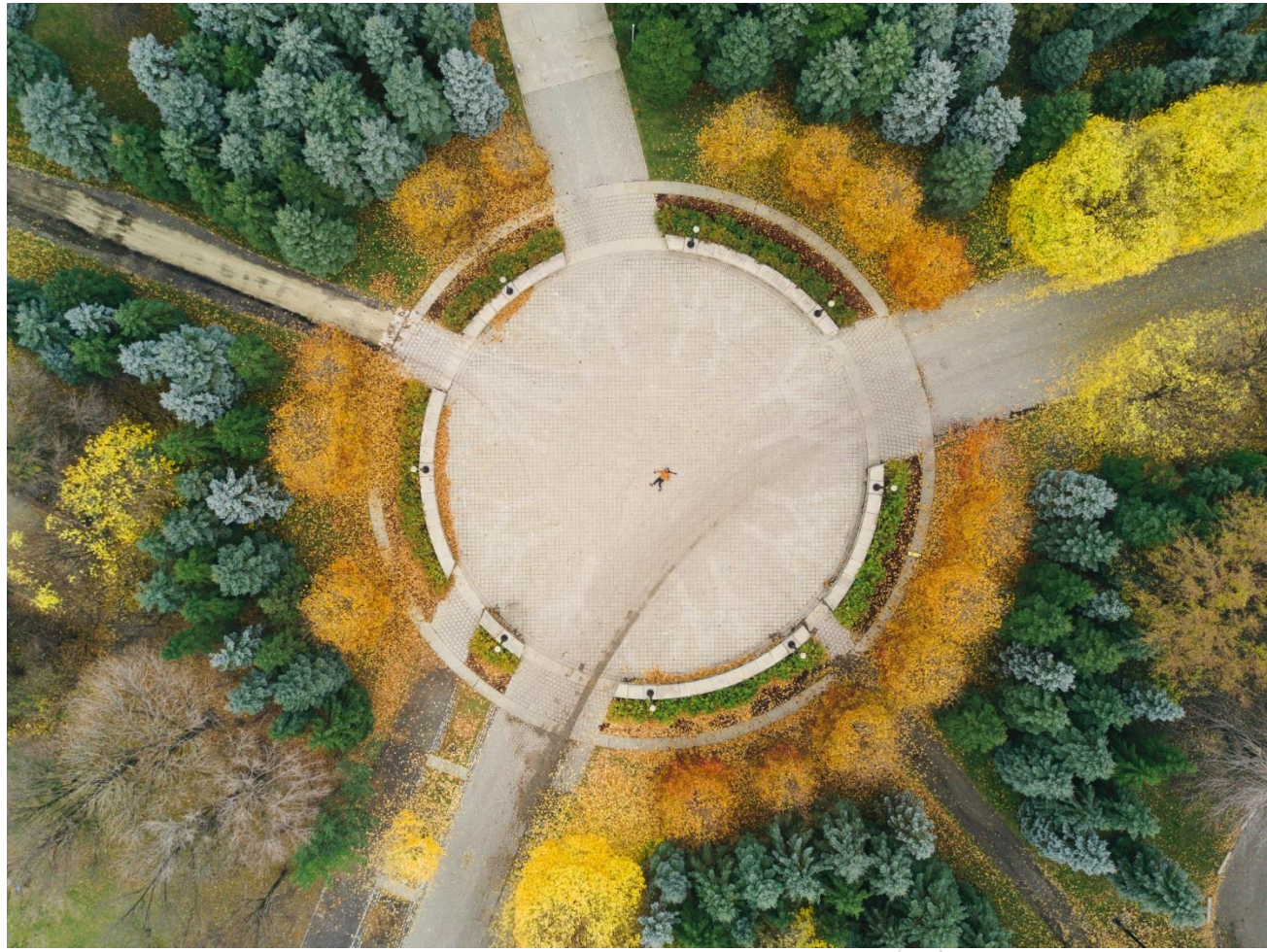





European Commission

# Aggregation methods

- Based on Average Values
- Based on Ranks
- Based on Pairwise comparison



# Arithmetic mean

The arithmetic mean of a list of n real numbers equals:

$$\frac{1}{n}\sum_{i=1}^{n} x_i \quad = \quad \frac{x_1 + x_2 + \cdots + x_n}{n}$$

This is the simplest, most obvious and most widespread aggregation method

**Perfect (and constant) substitutability** – underperformance in one component can be perfectly compensated by equivalent overperformance in another

# Geometric mean

The ***geometric mean*** of a list of *n positive* real numbers equals:

$$\sqrt[n]{\prod_{i=1}^{n} x_i} = \sqrt[n]{x_1 \times x_2 \times \cdots \times x_n}$$

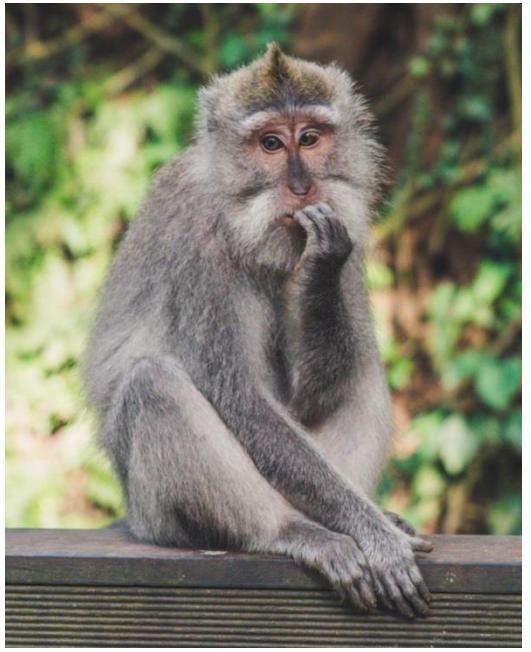
The first “less-compensatory” option

**Partial substitutability** – *unbalanced performance is always penalised* by the aggregation formula when compared to arithmetic aggregation

# The formula is the decision approach

Safe: 1
Yummy: 10

| |
|---|
| A.M.: **5.5** |
| G.M.: 3.2 |



Safe: 5
Yummy: 5

| |
|---|
| A.M.: 5.0 |
| G.M.: **5.0** |

Safe: 9
Yummy: 2

| |
|---|
| A.M.: **5.5** |
| G.M.: 4.2 |

# Geometric vs. Arithmetic mean: Implications

| | Sufficient Food | Sufficient to Drink | Safe Sanitation | Basic Needs (arithmetic) | Country *i's* improvement | Basic Needs (geometric) | Country *i's* improvement |
|---|---|---|---|---|---|---|---|
| Country *i* (t) | 10.0 | 8.6 | 1.4 | 6.7 | | 4.9 | |

# The two averages

Common features:

- ***Not robust to Outliers***
- ***Normalisation***
- ***Quantitative information needed and kept***
- ***"Good enough" correlation structure is necessary***
- ***Weights*** are ***interpreted as trade-offs***

Differences:

- ***Perfect substitutability*** vs. ***partial substitutability***
- ***Arithmetic mean*** is always ***greater than or equal to*** the equivalent ***geometric mean***

# Aggregation with an average

Before proceeding for a mean, check all the previous steps (normalisation, outliers, missing data, weights)

Remember: You need *quantitative* indicators

# Qualitative and quantitative together

Multi-criteria performance matrix

| | Criterion 1 (/20) | Criterion 2 (rating) | Criterion 3 (qual.) | Criterion 4 (Y/N) |
|---|---|---|---|---|
| Alternative 1 | 20 | 135 | G | Yes |
| Alternative 2 | 9 | 156 | B | Yes |
| Alternative 3 | 15 | 129 | VG | No |
| Alternative 4 | 9 | 146 | VB | No |
| Alternative 5 | 7 | 121 | G | Yes |
| … | … | … | … | … |

We need some method to compare, rank or evaluate the alternatives

# Methods based on ranks

Every criterion represents a voter, a point of view,
and determines a complete ranking

| | Criterion 1 (/20) | Criterion 2 (rating) | Criterion 3 (qual.) | Rank 1 | Rank 2 | Rank 3 |
|---|---|---|---|---|---|---|
| Alternative 1 | 20 | 135 | Good | 1 | 3 | 2.5 |
| Alternative 2 | 9 | 156 | Bad | 3.5 | 1 | 4 |
| Alternative 3 | 15 | 129 | Very Good | 2 | 4 | 1 |
| Alternative 4 | 9 | 146 | Very Bad | 3.5 | 2 | 5 |
| Alternative 5 | 7 | 121 | Good | 5 | 5 | 2.5 |

# Median rank

**3** units: **A, B, C**

**11** indicators/criteria

| **5** indicators | **4** indicators | **2** indicators |
|---|---|---|
| A | C | C |
| C | A | B |
| B | B | A |

| Ordered ranks |
|---|
| A: 11111**2**22233 |
| B:22333**3**33333 |
| C:11111**1**22222 |

| Ranking |
|---|
| C |
| A |
| B |

# Weighted median rank

**3** units: **A, B, C**

**6** indicators/ criteria **with (unequal) weights**

| Rank | Ind. 1 | Ind. 2 | Ind. 3 | Ind. 4 | Ind. 5 | Ind. 6 |
|---|---|---|---|---|---|---|
| | **0.05** | **0.25** | **0.30** | **0.10** | **0.05** | **0.25** |
| 1 | A | C | C | B | A | B |
| 2 | C | A | B | A | B | C |
| 3 | B | B | A | C | C | A |

| | Cumulative weight | | | |
|---|---|---|---|---|
| **Unit** | **0.10** | **0.25** | **0.50** | **0.75** |
| A | 1 | 2 | **3** | 3 |
| B | 1 | 1 | **2** | 3 |
| C | 1 | 1 | **1** | 2 |

# Methods based on pairwise comparisons

| | Criterion 1 (/20) | Criterion 2 (rating) | Criterion 3 (qual.) | Criterion 4 (Y/N) | … |
|---|---|---|---|---|---|
| Alternative 1 | 20 | 135 | G | Yes | … |
| Alternative 2 | 9 | 156 | B | Yes | … |
| Alternative 3 | 15 | 129 | VG | No | … |
| Alternative 4 | 9 | 146 | VB | No | … |
| Alternative 5 | 7 | 121 | G | Yes | … |
| … | … | … | … | … | … |

Compare alternatives using the original values, all criteria simultaneously

Example: Alternative 1 is better than Alternative 5

# Outranking Matrix – Concordance value

Step 1 – Raw data, Weights & Orientation

| | Index | | | |
|---|---|---|---|---|
| | Ind 01 | Ind 02 | Ind 03 | Ind04 |
| Orientation | -1 | -1 | -1 | -1 |
| Weights | 0.25 | 0.25 | 0.25 | 0.25 |
| MT | 74.9 | Good | 205.0 | 14.7 |
| NL | 75.8 | Acceptable | 47.9 | 11.6 |
| PL | 58.9 | Excellent | 87.9 | 5.8 |
| PT | 124.3 | Very good | 445.5 | 18.8 |
| RO | 38.5 | Very Poor | 207.6 | 7.3 |
| SE | 39.0 | None | 17.7 | 7.0 |
| SI | 66.5 | Outstanding | 350.9 | 5.6 |
| SK | 56.7 | Poor | 87.8 | 9.2 |

# Outranking Matrix – Concordance value

| | Index | | | |
|---|---|---|---|---|
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| Orientation | -1 | -1 | -1 | -1 |
| Weights | 0.25 | 0.25 | 0.25 | 0.25 |
| MT | 74.9 | Good | 205.0 | 14.7 |
| NL | 75.8 | Acceptable | 47.9 | 11.6 |
| PL | 58.9 | Excellent | 87.9 | 5.8 |
| PT | 124.3 | Very good | 445.5 | 18.8 |
| RO | 38.5 | Very Poor | 207.6 | 7.3 |
| SE | 39.0 | None | 17.7 | 7.0 |
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Step 1 – Raw data & Weights & Orientation

Step 2 – Concordance value

**For *n* countries, there are *n (n-1)* pairwise comparisons to be made**

Example:
MT versus NL = 0.25
NL versus MT = 0.75

Sum = 1.00

# Outranking Matrix – Concordance value

| | Index | | | |
|---|---|---|---|---|
| | Ind 01 | Ind 02 | Ind 03 | Ind04 |
| Orientation | -1 | -1 | -1 | -1 |
| Weights | 0.25 | 0.25 | 0.25 | 0.25 |
| MT | 74.9 | Good | 205.0 | 14.7 |
| NL | 75.8 | Acceptable | 47.9 | 11.6 |
| PL | 58.9 | Excellent | 87.9 | 5.8 |
| PT | 124.3 | Very good | 445.5 | 18.8 |
| RO | 38.5 | Very Poor | 207.6 | 7.3 |
| SE | 39.0 | None | 17.7 | 7.0 |
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Step 1 – Raw data & Weights & Orientation

Step 2 – Concordance value

For *n* countries, there are ***n (n-1)*** **pairwise comparisons** to be made

Example:
MT versus PT = 1.00
PT versus MT = 0.00

Sum = 1.00 → **Robust pair**

# Outranking Matrix – Construction

Step 1 – Raw data & Weights & Orientation

Step 2 – Concordance value

Step 3 – **Outranking matrix**

All concordance values are entered in the outranking matrix.
(entries above and below the diagonal sum up to 1.0)

| | MT | NL | PL | PT | RO | SE | SI | SK |
|---|---|---|---|---|---|---|---|---|
| MT | 0.00 | 0.25 | 0.25 | 1.00 | 0.25 | 0.00 | 0.50 | 0.00 |
| NL | 0.75 | 0.00 | 0.50 | 1.00 | 0.25 | 0.00 | 0.50 | 0.25 |
| PL | 0.75 | 0.50 | 0.00 | 0.75 | 0.50 | 0.25 | 0.75 | 0.50 |
| PT | 0.00 | 0.00 | 0.25 | 0.00 | 0.00 | 0.00 | 0.25 | 0.00 |
| RO | 0.75 | 0.75 | 0.50 | 1.00 | 0.00 | 0.25 | 0.75 | 0.75 |
| SE | 1.00 | 1.00 | 0.75 | 1.00 | 0.75 | 0.00 | 0.75 | 1.00 |
| SI | 0.50 | 0.50 | 0.25 | 0.75 | 0.25 | 0.25 | 0.00 | 0.25 |
| SK | 1.00 | 0.75 | 0.50 | 1.00 | 0.25 | 0.00 | 0.75 | 0.00 |

MT versus NL = 0.25
NL versus MT = 0.75

MT versus PT = 1.00
PT versus MT = 0.00

# The Copeland Score

***Victories (+1)*** minus ***Defeats (-1)*** (***ties don‘t count***)

**Outranking matrix**

| | MT | NL | PL | PT | RO | SE | SI | SK |
|---|---|---|---|---|---|---|---|---|
| MT | 0.00 | 0.25 | 0.25 | 1.00 | 0.25 | 0.00 | 0.50 | 0.00 |
| NL | 0.75 | 0.00 | 0.50 | 1.00 | 0.25 | 0.00 | 0.50 | 0.25 |
| PL | 0.75 | 0.50 | 0.00 | 0.75 | 0.50 | 0.25 | 0.75 | 0.50 |
| PT | 0.00 | 0.00 | 0.25 | 0.00 | 0.00 | 0.00 | 0.25 | 0.00 |
| RO | 0.75 | 0.75 | 0.50 | 1.00 | 0.00 | 0.25 | 0.75 | 0.75 |
| SE | 1.00 | 1.00 | 0.75 | 1.00 | 0.75 | 0.00 | 0.75 | 1.00 |
| SI | 0.50 | 0.50 | 0.25 | 0.75 | 0.25 | 0.25 | 0.00 | 0.25 |
| SK | 1.00 | 0.75 | 0.50 | 1.00 | 0.25 | 0.00 | 0.75 | 0.00 |

| | Wins | Defeats | Total | Rank |
|---|---|---|---|---|
| SE | 7 | 0 | 7 | 1 |
| RO | | | | |
| SK | | | | |
| PL | | | | |
| NL | | | | |
| SI | | | | |
| MT | | | | |
| PT | | | | |

# The Copeland Score

**Outranking matrix**

| | MT | NL | PL | PT | RO | SE | SI | SK |
|---|---|---|---|---|---|---|---|---|
| MT | 0.00 | 0.25 | 0.25 | 1.00 | 0.25 | 0.00 | 0.50 | 0.00 |
| NL | 0.75 | 0.00 | 0.50 | 1.00 | 0.25 | 0.00 | 0.50 | 0.25 |
| PL | 0.75 | 0.50 | 0.00 | 0.75 | 0.50 | 0.25 | 0.75 | 0.50 |
| PT | 0.00 | 0.00 | 0.25 | 0.00 | 0.00 | 0.00 | 0.25 | 0.00 |
| RO | 0.75 | 0.75 | 0.50 | 1.00 | 0.00 | 0.25 | 0.75 | 0.75 |
| SE | 1.00 | 1.00 | 0.75 | 1.00 | 0.75 | 0.00 | 0.75 | 1.00 |
| SI | 0.50 | 0.50 | 0.25 | 0.75 | 0.25 | 0.25 | 0.00 | 0.25 |
| SK | 1.00 | 0.75 | 0.50 | 1.00 | 0.25 | 0.00 | 0.75 | 0.00 |

| | Wins | Defeats | Total | Rank |
|---|---|---|---|---|
| SE | 7 | 0 | 7 | 1 |
| RO | 5 | 1 | 4 | 2 |
| SK | 4 | 2 | 2 | 3 |
| PL | 3 | 1 | 2 | 4 |
| NL | 2 | 3 | -1 | 5 |
| SI | 1 | 4 | -3 | 6 |
| MT | 1 | 5 | -4 | 7 |
| PT | 0 | 6 | -6 | 8 |

This method is in the COIN tool and the COINr package

# Summary of methods on Pairwise Comparison

- Fully ***non-compensatory*** approach;
- ***only weights and orientation*** are ***required*** to obtain the ranking of alternatives;
- ***weights*** represent exactly ***the importance*** *of the indicator*;
- no impact of outliers;
- no need for data normalisation
- no need for ”good” correlation structure;
- can be used both with continuous and categorical variables;
- computationally more demanding than standard averages;
- poor with small number of units;
- software available for Copeland (send an email to JRC-COIN): Excel, R, Matlab


Sources: Athanasoglou (2015) , Tarjan (1972), Van Zuylen, and Williamson (2009), Munda and Nardo (2009)

# Thank you

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https://knowledge4policy.ec.europa.eu/composite-indicators_en





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