The European Commission's science and knowledge service

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Step 6: Aggregation

Marcos Domínguez-Torreiro PhD

Giulio Caperna PhD

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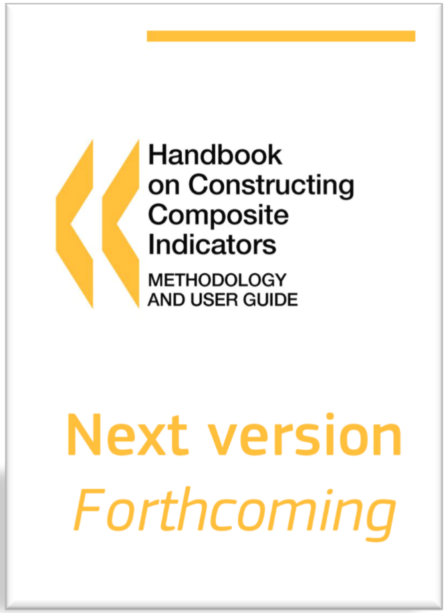
Ten steps



- **Step 1** Developing the framework
- **Step 2** Selection of indicators
- **Step 3** Data treatment
- **Step 4** Normalisation
- **Step 5** Weighting
- **Step 6** Aggregation
- **Step 7** Statistical coherence
- **Step 8** Robustness & sensitivity
- **Step 9** Back to the data
- **Step 10** Visualisation

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Aggregation methods

- Based on Average Scores
  1. Arithmetic Mean
  2. Geometric Mean
- Based on Ranks
  3. Median rank
  4. Majority
  5. Borda’s Count
- Based on Pairwise Comparisons
  6. Condorcet
  7. Kemeny
  8. Arrow – Raynaud
  9. Copeland

Aggregation methods

- Based on Average Scores
  1. Arithmetic Mean
  2. Geometric Mean

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Arithmetic mean (additive aggregation)

The ***arithmetic mean*** of a list of *n* real numbers equals:

$$\frac{1}{n}\sum_{i=1}^{n} x_i = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

This is the simplest, most obvious and most widespread aggregation method

Perfect (and constant) substitutability – *the impact on the aggregate score* of *a unit increase (decrease) in the level of any indicator is the same*, i.e. underperformance in one component can be perfectly compensated by equivalent overperformance in another

Arithmetic mean in CIs

The score corresponding to the 4th pillar of the GTCI index in country *i* is calculated as The arithmetic average of sub-pillars 4.1 and 4.2:

Sustainability score = 37.04
Lifestyle score = 59.60

Retain pillar score = ½ (37.04+59.60) = 48.32

4	**RETAIN**	**48.32**	**70**
4.1	Sustainability	37.04	78
4.1.1	Pension system	37.37	56
4.1.2	Taxation	43.58	73
4.1.3	Brain retention	30.16	94
4.2	Lifestyle	59.60	64
4.2.1	Environmental performance	69.58	57
4.2.2	Personal safety	62.00	56
4.2.3	Physician density	14.55	78
4.2.4	Sanitation	92.27	57

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Aggregation methods

- Based on Average Scores
  1. Arithmetic Mean
  2. Geometric Mean

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Geometric mean (multiplicative aggregation)

The ***geometric mean*** of a list of *n* *positive* real numbers equals:

$$\sqrt[n]{\prod_{i=1}^{n} x_i} = \sqrt[n]{x_1 \times x_2 \times \cdots \times x_n}$$

Partial substitutability – compensation of low performance on some indicators by high performance on others is possible only partially, i.e. *unbalanced performance is always penalised* by the aggregation formula when compared to arithmetic aggregation

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Geometric mean in CIs

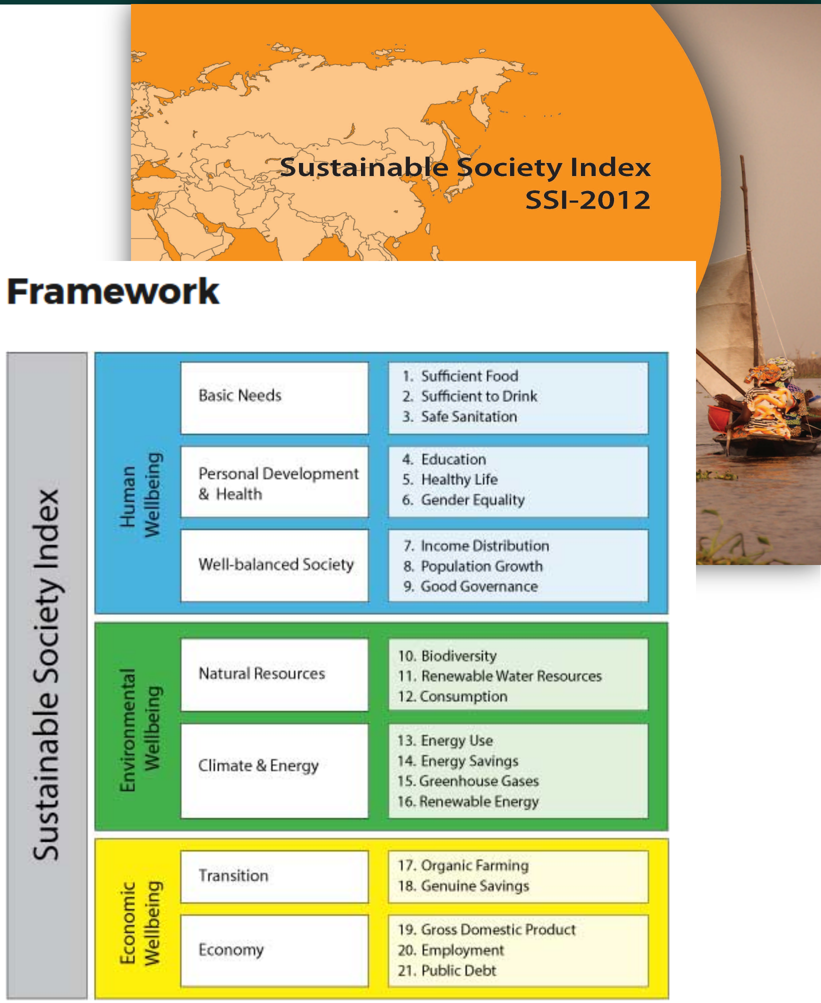
Basic Needs indicators - Country *i*	value
% people with sufficient food	92
% people with safe drinking water	79
% people with safe sanitation	17

$$\text{Sufficient Food} = \frac{92}{10} = 9.2$$

$$\text{Sufficient to Drink} = \frac{79}{10} = 7.9$$

$$\text{Safe Sanitation} = \frac{17}{10} = 1.7$$

$$\text{Basic Needs Pillar} = \sqrt[3]{9.2 \cdot 7.9 \cdot 1.7} = 4.98$$



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Rational choice and risk aversion: What would the monkey have for lunch today?" (arithmetic vs. geometric aggregation)

Safe: 1
Yummy: 10

A.M.:	**5.5**
G.M.:	3.2

Arithmetic mean results in more tolerance towards extreme values (high-risk-low-taste and vice versa); therefore, if not willing to trade off very high (low) risk levels for very good (bad) taste, geometric mean can guide you to a more conservative decision!

Safe: 5
Yummy: 5

A.M.:	5.0
G.M.:	**5.0**

Safe: 9
Yummy: 2

A.M.:	**5.5**
G.M.:	4.2

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Geometric vs. Arithmetic mean: Implications for policy making

	I.1. Sufficient Food	I.2. Sufficient to Drink	I.3. Safe Sanitation	I. Basic Needs Pillar Score (arithmetic)	Country *i's* improvement in *t+1*	I. Basic Needs Pillar Score (geometric)	Country *i's* improvement in *t+1*
Country *i* (t)	10.0	8.6	1.4	6.7		4.9	

Geometric vs. Arithmetic mean: Implications for policy making

	I.1. Sufficient Food	I.2. Sufficient to Drink	I.3. Safe Sanitation	I. Basic Needs Pillar Score (arithmetic)	Country *i's* improvement in *t+1*	I. Basic Needs Pillar Score (geometric)	Country *i's* improvement in *t+1*
Country *i* (t)	10.0	8.6	1.4	6.7		4.9	
Following year							
[a.1] Country *i* (t+1)	10.0	**9.6** **+1**	1.4	7.0	4.5%	5.1	4.1%
[a.2] Country *i* (t+1)	10.0	8.6	**2.4** **+1**	7.0	4.5%	5.9	20.4%
[b.1] Country *i* (t+1)	10.0	**9.6** **+1**	**0.4** **-1**	6.7	0%	3.4	-31.1%
[b.2] Country *i* (t+1)	10.0	**7.6** **-1**	**2.4** **+1**	6.7	0%	5.7	15.7%

With geometric mean:

1) *poor performance in one indicator* ***is not perfectly compensated*** *by good performance in another*;

2) ***improvements in the weaker dimensions are encouraged***;

3) ***unbalanced situations are penalised / balance is rewarded in the aggregate score***

Note: Using unequal weights

For a sequence of positive weights w_i, with $\Sigma w_i = 1$, the ***weighted arithmetic mean*** equals:

$$\sum_{i=1}^{n} w_i x_i \quad = \quad w_1 x_1 + w_2 x_2 + \cdots + w_n x_n$$

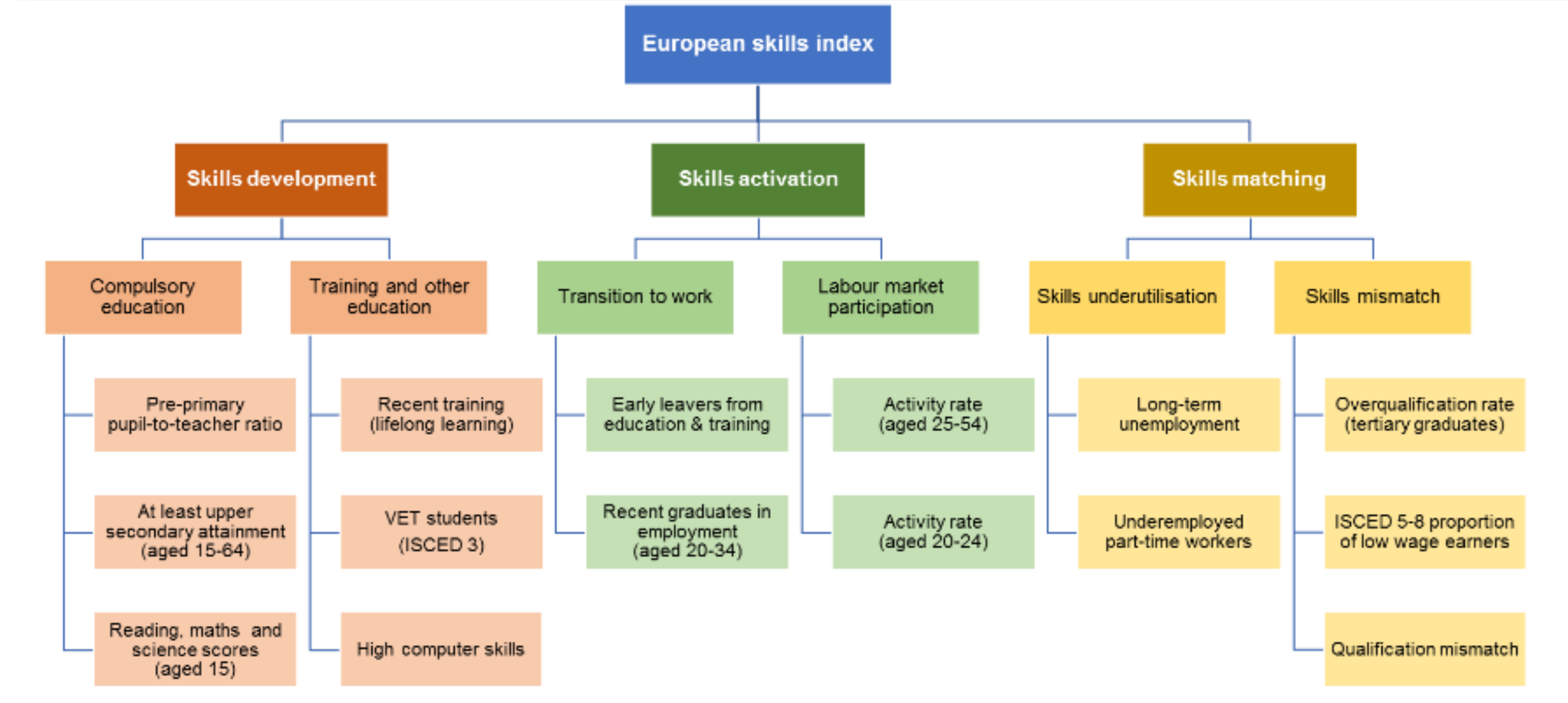
For a sequence of positive weights w_i, with $\Sigma w_i = 1$, the ***weighted geometric mean*** equals:

$$\prod_{i=1}^{n} {x_i}^{w_i} \quad = {x_1}^{w_1} \times {x_2}^{w_2} \times \cdots \times {x_n}^{w_n}$$

Note: Hybrid aggregation in CIs

CIs using different aggregation functions at different levels of aggregation (mixed approach)

European Skills Index (CEDEFOP)	Arithmetic average within dimensions	Geometric average across dimensions



In a nutshell

Methods based on average scores share common features…

- ***Interval level (quantitative) variables*** and ***"good enough" correlation structure are required*** to compute meaningful averages
- ***Interval level information is kept*** during the aggregation process (i.e. output interpreted as ***quantitative scores***, not just as ordered sequence of units)
- ***Normalisation*** of indicators required prior to aggregation
- Averages ***highly influenced by outliers*** in the data
- ***Weights*** in the aggregation formulas are to be ***interpreted as trade-offs*** (i.e. how much an advantage on one indicator can offset a disadvantage on another), and not as importance coefficients (i.e. "the most important indicator is precisely the one with the highest weight")

… but also significant differences

- ***Perfect (and constant) substitutability*** (arithmetic) vs. ***partial substitutability*** penalising unbalanced performance (geometric)
- ***Arithmetic mean*** is always ***greater than or equal to*** the equivalent ***geometric mean***

A word of caution regarding the use of averages

- ✓ Averages should be avoided when ***correlation structure is not good/strong enough*** (0.4-0.8): aggregate results highly sensitive to underlying methodological and conceptual assumptions, hampering meaningful interpretations
- ✓ At times, treating ***outliers*** is counterproductive: outlying values could be key to the ongoing analysis and should be kept in the data set!
- ✓ The data set might include ***not only quantitative but also qualitative information***

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Example: meaningful outliers and weak correlations

The ESRB Dashboard and Heatmaps - Indicators of systemic risk in the EU financial system

			average	st.d.	min	max	skew	kurt
MACRO	Current real GDP growth	2.1	0.10	2.60	-7.92	5.99	-0.12	1.07
	Domestic credit-to-GDP gap	2.2	-6.77	6.06	-21.88	0.41	-1.11	0.30
	Current account balance-to-GDP ratio	2.3	0.02	4.09	-9.89	10.27	0.65	0.18
	Rate of unemployment	2.4	10.83	5.17	4.15	26.22	1.35	1.83
FISCAL	Forecast government debt-to-GDP ratio	2.5	67.83	35.26	6.25	170.32	0.65	0.30
	Forecast government deficit-to-GDP ratio	2.6	4.02	2.83	0.15	13.38	0.93	0.91
	Credit default swap premia on sovereign debt	2.7	574.16	1836.22	18.63	12447.07	**5.57**	**31.71**
	Annual sovereign debt redemptions as a share of GDP	2.8	14.51	11.07	0.00	47.37	0.81	-0.09
HH	Households' debt-to-gross disposable income ratio	2.9	104.67	61.58	36.88	268.92	1.23	0.86
	Estimates of the over/undervaluation of residential property prices	3.1.a.	2.58	11.85	-12.67	28.39	0.62	-0.93
	Share of foreign currency loans on total loans to non-MFIs	3.2a	18.81	25.58	0.28	89.45	1.55	0.97
	MFI lending to HH (annual growth rates) NEW	n.a.2	0.66	4.75	-16.69	11.12	-0.68	1.92
NFC	Non-financial corporations' debt-to-GDP ratio	2.13	115.88	74.87	0.00	555.04	**2.74**	**14.13**
	MFI lending to NFC (annual growth rates) NEW	n.a.1	0.50	5.02	-10.79	14.01	0.23	-0.41
MFIs	Share of central bank funding in credit institutions liabilities	4.5	4.60	7.28	0.00	34.78	**2.58**	**7.24**
	MFI's exposure to domestic sovereign (share of total assets) NEW	n.a.3	0.08	0.06	0.00	0.23	0.92	-0.11
	Banking sector leverage NEW	n.a.4	16.16	7.22	4.98	50.46	1.35	**4.87**
	Loan to deposit ratio NEW	n.a.5	1.31	0.47	0.61	2.97	1.93	**4.43**

Note: European Systemic Risk Board, raw data, pooled dataset: 2013Q3, 2012Q4, 2012 Q3, 2011 Q4 (four time-points x 28 countries)

Outliers in some indicators: problematic when analysing the correlation structure

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Example: meaningful outliers and weak correlations

	Indicator	Code	average	st.d.	min	max	skew	kurt
MACRO	Current real GDP growth	2.1	0.10	2.60	-7.92	5.99	-0.12	1.07
MACRO	Domestic credit-to-GDP gap	2.2	-6.77	6.06	-21.88	0.41	-1.11	0.30
MACRO	Current account balance-to-GDP ratio	2.3	0.02	4.09	-9.89	10.27	0.65	0.18
MACRO	Rate of unemployment	2.4	10.83	5.17	4.15	26.22	1.35	1.83
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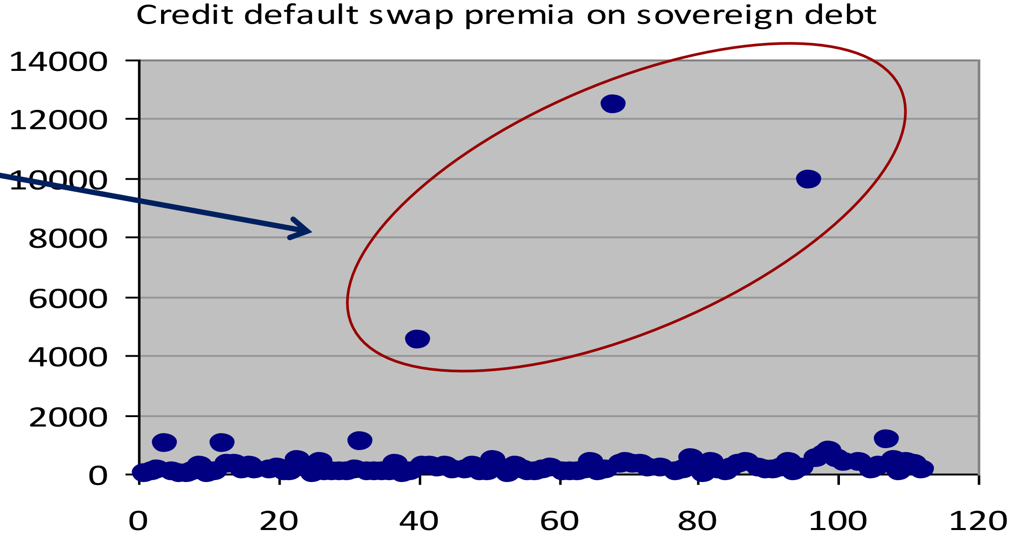
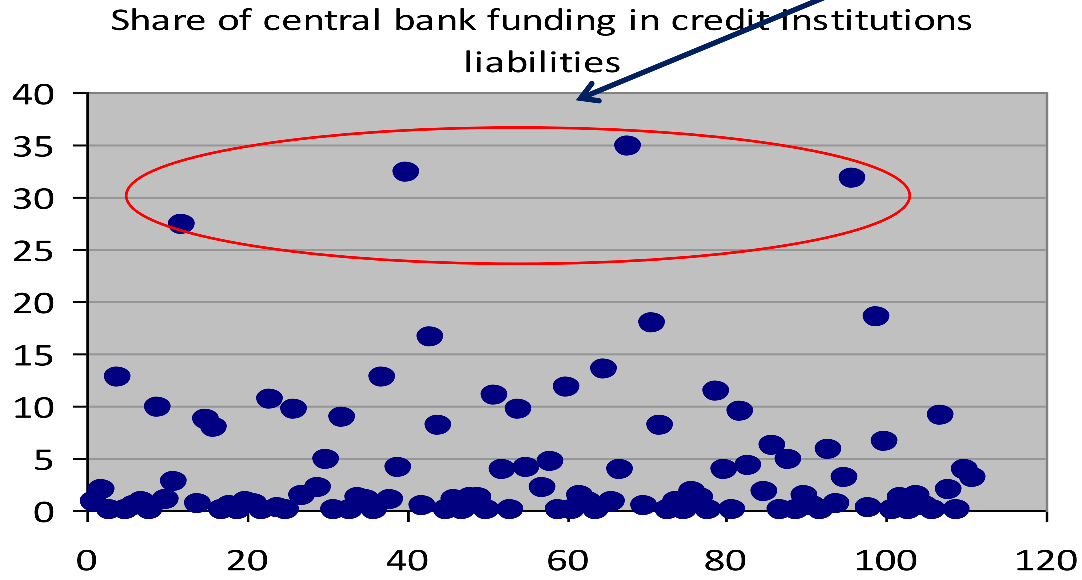
FISCAL	Annual sovereign debt redemptions as a share of GDP	2.8	14.51	11.07	0.00	47.37	0.81	-0.09
HH	Households' debt-to-gross disposable income ratio	2.9	104.67	61.58	36.88	268.92	1.23	0.86
HH	Estimates of the over/undervaluation of residential property prices	3.1.a.	2.58	11.85	-12.67	28.39	0.62	-0.93
HH	Share of foreign currency loans on total loans to non-MFIs	3.2a	18.81	25.58	0.28	89.45	1.55	0.97
HH	MFI lending to HH (annual growth rates) NEW	n.a.2	0.66	4.75	-16.69	11.12	-0.68	1.92
NFC	Non-financial corporations' debt-to-GDP ratio	2.13	115.88	74.87	0.00	555.04	2.74	14.13
NFC	MFI lending to NFC (annual growth rates) NEW	n.a.1	0.50	5.02	-10.79	14.01	0.23	-0.41
MFIs	Share of central bank funding in credit institutions liabilities	4.5	4.60	7.28	0.00	34.78	2.58	7.24
MFIs	MFI's exposure to domestic sovereign (share of total assets) NEW	n.a.3	0.08	0.06	0.00	0.23	0.92	-0.11
MFIs	Banking sector leverage NEW	n.a.4	16.16	7.22	4.98	50.46	1.35	4.87
MFIs	Loan to deposit ratio NEW	n.a.5	1.31	0.47	0.61	2.97	1.93	4.43





Outliers need to be treated before carrying out correlation analysis and using averages for aggregation.

However, the ***aim of the ESRB is to monitor and analyse "extreme" behaviour***… and that precludes getting read of outliers in the data set!

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Example: meaningful outliers and weak correlations

	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3.1.a.	3.2a	n.a.2	2.13	n.a.1	4.5	n.a.3	n.a.4	n.a.5
2.1	[1 ,1]	[-0.2 ,0.1]	[0.1 ,0.2]	[-0.4 ,-0.2]	[-0.7 ,-0.6]	[-0.6 ,-0.2]	[-0.7 ,-0.5]	[-0.6 ,-0.4]	[-0.4 ,-0.2]	[-0.3 ,0.3]	[0.4 ,0.6]	[0 ,0.2]	[-0.3 ,-0.1]	[0 ,0.4]	[-0.7 ,-0.5]	[-0.2 ,-0.1]	[-0.6 ,-0.4]	[-0.2 ,-0.1]
2.2	[-0.2 ,0.1]	[1 ,1]	[-0.3 ,0]	[-0.6 ,-0.1]	[-0.2 ,0.1]	[-0.5 ,-0.3]	[-0.1 ,0]	[0 ,0.3]	[-0.5 ,-0.5]	[0.1 ,0.5]	[-0.1 ,0]	[0.2 ,0.5]	[-0.5 ,-0.4]	[0.5 ,0.6]	[-0.5 ,-0.1]	[0.3 ,0.6]	[-0.2 ,-0.1]	[-0.4 ,-0.3]
2.3	[0.1 ,0.2]	[-0.3 ,0]	[1 ,1]	[-0.6 ,-0.2]	[-0.4 ,-0.2]	[-0.5 ,-0.3]	[-0.6 ,-0.4]	[-0.2 ,0]	[0.4 ,0.5]	[-0.4 ,-0.3]	[-0.3 ,-0.2]	[0.1 ,0.1]	[0.2 ,0.4]	[-0.1 ,-0.1]	[-0.4 ,-0.2]	[-0.5 ,-0.2]	[0.1 ,0.4]	[0.3 ,0.4]
2.4	[-0.4 ,-0.2]	[-0.6 ,-0.1]	[-0.6 ,-0.2]	[1 ,1]	[0.2 ,0.5]	[0.6 ,0.7]	[0.1 ,0.8]	[0 ,0.2]	[-0.1 ,0.1]	[-0.1 ,0.1]	[0 ,0.2]	[-0.4 ,-0.1]	[-0.1 ,0.1]	[-0.4 ,-0.3]	[0.3 ,0.7]	[0.1 ,0.3]	[-0.2 ,0.3]	[0 ,0.1]
2.5	[-0.7 ,-0.6]	[-0.2 ,0.1]	[-0.4 ,-0.2]	[0.2 ,0.5]	[1 ,1]	[0.5 ,0.7]	[0.6 ,0.6]	[0.6 ,0.7]	[0.1 ,0.2]	[-0.5 ,0.2]	[-0.4 ,-0.3]	[-0.4 ,-0.3]	[0 ,0.1]	[-0.5 ,-0.2]	[0.8 ,0.8]	[0 ,0.1]	[0.4 ,0.6]	[-0.1 ,-0.1]
2.6	[-0.6 ,-0.2]	[-0.5 ,-0.3]	[-0.5 ,-0.3]	[0.6 ,0.7]	[0.5 ,0.7]	[1 ,1]	[0.4 ,0.6]	[0.2 ,0.4]	[0.1 ,0.2]	[-0.3 ,0.1]	[-0.3 ,0]	[-0.3 ,-0.2]	[-0.1 ,0.1]	[-0.5 ,-0.2]	[0.6 ,0.8]	[0.1 ,0.2]	[-0.1 ,0.4]	[-0.1 ,0.1]
2.7	[-0.7 ,-0.5]	[-0.1 ,0]	[-0.6 ,-0.4]	[0.1 ,0.8]	[0.6 ,0.6]	[0.4 ,0.6]	[1 ,1]	[0.2 ,0.6]	[-0.2 ,0.1]	[-0.4 ,0.2]	[-0.1 ,0.1]	[-0.5 ,-0.3]	[0 ,0.2]	[-0.6 ,-0.3]	[0.7 ,0.8]	[0 ,0.2]	[-0.2 ,0.5]	[-0.2 ,-0.1]
2.8	[-0.6 ,-0.4]	[0 ,0.3]	[-0.2 ,0]	[0 ,0.2]	[0.6 ,0.7]	[0.2 ,0.4]	[0.2 ,0.6]	[1 ,1]	[-0.1 ,0.3]	[-0.2 ,0.5]	[-0.4 ,-0.3]	[-0.3 ,0]	[0.1 ,0.2]	[-0.2 ,0.1]	[0.4 ,0.4]	[0 ,0.3]	[0.2 ,0.6]	[-0.2 ,-0.1]
2.9	[-0.4 ,-0.2]	[-0.5 ,-0.5]	[0.4 ,0.5]	[-0.1 ,0.1]	[0.1 ,0.2]	[0.1 ,0.2]	[-0.2 ,0.1]	[-0.1 ,0.3]	[1 ,1]	[-0.5 ,-0.1]	[-0.4 ,-0.3]	[-0.1 ,0.1]	[0.5 ,0.5]	[-0.2 ,0]	[0.1 ,0.2]	[-0.6 ,-0.5]	[0.4 ,0.6]	[0.5 ,0.6]
3.1.a.	[-0.3 ,0.3]	[0.1 ,0.5]	[-0.4 ,-0.3]	[-0.1 ,0.1]	[-0.5 ,0.2]	[-0.3 ,0.1]	[-0.4 ,0.2]	[-0.2 ,0.5]	[-0.5 ,-0.1]	[1 ,1]	[-0.4 ,0]	[0 ,0.7]	[0 ,0.3]	[0.2 ,0.4]	[-0.4 ,0.3]	[-0.1 ,0.1]	[0.2 ,0.4]	[-0.2 ,0.3]
3.2a	[0.4 ,0.6]	[-0.1 ,0]	[-0.3 ,-0.2]	[0 ,0.2]	[-0.4 ,-0.3]	[-0.3 ,0]	[-0.1 ,0.1]	[-0.4 ,-0.3]	[-0.4 ,-0.3]	[-0.4 ,0]	[1 ,1]	[-0.4 ,-0.4]	[-0.4 ,-0.3]	[0 ,0.3]	[-0.3 ,-0.3]	[0.1 ,0.1]	[-0.5 ,-0.4]	[-0.2 ,-0.1]
n.a.2	[0 ,0.2]	[0.2 ,0.5]	[0.1 ,0.1]	[-0.4 ,-0.1]	[-0.4 ,-0.3]	[-0.3 ,-0.2]	[-0.5 ,-0.3]	[-0.3 ,0]	[-0.1 ,0.1]	[0 ,0.7]	[-0.4 ,-0.4]	[1 ,1]	[-0.1 ,0.1]	[0.2 ,0.6]	[-0.5 ,-0.2]	[0 ,0.1]	[-0.1 ,0]	[-0.1 ,0]
2.13	[-0.3 ,-0.1]	[-0.5 ,-0.4]	[0.2 ,0.4]	[-0.1 ,0.1]	[0 ,0.1]	[-0.1 ,0.1]	[0 ,0.2]	[0.1 ,0.2]	[0.5 ,0.5]	[0 ,0.3]	[-0.4 ,-0.3]	[-0.1 ,0.1]	[1 ,1]	[-0.4 ,-0.2]	[0.1 ,0.2]	[-0.6 ,-0.6]	[0.1 ,0.4]	[0.1 ,0.2]
n.a.1	[0 ,0.4]	[0.5 ,0.6]	[-0.1 ,-0.1]	[-0.4 ,-0.3]	[-0.5 ,-0.2]	[-0.5 ,-0.2]	[-0.6 ,-0.3]	[-0.2 ,0.1]	[-0.2 ,0]	[0.2 ,0.4]	[0 ,0.3]	[0.2 ,0.6]	[-0.4 ,-0.2]	[1 ,1]	[-0.6 ,-0.3]	[0 ,0.4]	[-0.3 ,0.1]	[-0.3 ,0]
4.5	[-0.7 ,-0.5]	[-0.5 ,-0.1]	[-0.4 ,-0.2]	[0.3 ,0.7]	[0.8 ,0.8]	[0.6 ,0.8]	[0.7 ,0.8]	[0.4 ,0.4]	[0.1 ,0.2]	[-0.4 ,0.3]	[-0.3 ,-0.3]	[-0.5 ,-0.2]	[0.1 ,0.2]	[-0.6 ,-0.3]	[1 ,1]	[0 ,0.1]	[0.2 ,0.5]	[0 ,0.1]
n.a.3	[-0.2 ,-0.1]	[0.3 ,0.6]	[-0.5 ,-0.2]	[0.1 ,0.3]	[0 ,0.1]	[0.1 ,0.2]	[0 ,0.2]	[0 ,0.3]	[-0.6 ,-0.5]	[-0.1 ,0.1]	[0.1 ,0.1]	[0 ,0.1]	[-0.6 ,-0.6]	[0 ,0.4]	[0 ,0.1]	[1 ,1]	[-0.4 ,-0.3]	[-0.3 ,-0.3]
n.a.4	[-0.6 ,-0.4]	[-0.2 ,-0.1]	[0.1 ,0.4]	[-0.2 ,0.3]	[0.4 ,0.6]	[-0.1 ,0.4]	[-0.2 ,0.5]	[0.2 ,0.6]	[0.4 ,0.6]	[0.2 ,0.4]	[-0.5 ,-0.4]	[-0.1 ,0]	[0.1 ,0.4]	[-0.3 ,0.1]	[0.2 ,0.5]	[-0.4 ,-0.3]	[1 ,1]	[0.2 ,0.4]
n.a.5	[-0.2 ,-0.1]	[-0.4 ,-0.3]	[0.3 ,0.4]	[0 ,0.1]	[-0.1 ,-0.1]	[-0.1 ,0.1]	[-0.2 ,-0.1]	[-0.2 ,-0.1]	[0.5 ,0.6]	[-0.2 ,0.3]	[-0.2 ,-0.1]	[-0.1 ,0]	[0.1 ,0.2]	[-0.3 ,0]	[0 ,0.1]	[-0.3 ,-0.3]	[0.2 ,0.4]	[1 ,1]

Notes: raw data (without outliers), pooled dataset: 2013Q3, 2012Q4, 2012 Q3, 2011 Q4, correlations less than 0.38 are not significant

Weak correlation structure: poor (or negative) correlations / correlation structure changing over time

- Most bivariate correlations are not significant at any of the 4 time-points
- No bivariate correlation is significant at all four time points
- Presence of significantly negative correlations

Example: Quantitative and qualitative indicators

Multi-criteria performance matrix

	Criterion 1 (/20)	Criterion 2 (rating)	Criterion 3 (qual.)	Criterion 4 (Y/N)
Alternative 1	20	135	G	Yes
Alternative 2	9	156	B	Yes
Alternative 3	15	129	VG	No
Alternative 4	9	146	VB	No
Alternative 5	7	121	G	Yes
…	…	…	…	…

Challenge: how to rank or evaluate the units (alternatives) according to a heterogeneous set of indicators (criteria)?

Social Choice Theory and Multi-Criteria Analysis

1	Ramon Llull
2	Nicolas de Condorcet
3	Nicholas of Kues
4	Jean-Charles, Chevalier de Borda

The problem as studied by ***Social Choice***:

- Election: voters selecting candidates from a pool
- Each voter ranking candidates according to his/her preferences
- Point of contention: what voting/aggregation of preferences system would select the canditate that « best » represents social preferences?

Similar problem in ***Multi-Criteria Analysis***:

- Candidates ↔ Units/Alternatives
- Voters ↔ Indicators/Criteria

Key assumptions:

- Indicators not expected to be related and dependent; indicators may be independent and even opposite (i.e. poor or even negative correlations among indicators)
- Usually, there is no optimal alternative maximising all the criteria at the same time; therefore compromise solutions have to be found to select the best option

Aggregation methods

- Based on Ranks

3. Median rank
4. Majority
5. Borda's Count

Methods based on ranks

Each indicator represents a "criterion" or "point of view"
and ***determines a complete ranking of the units*** (alternatives)

	Criterion 1 (/20)	Criterion 2 (rating)	Criterion 3 (qual.)	Rank 1	Rank 2	Rank 3
Alternative 1	20	135	Good	1	3	2.5
Alternative 2	9	156	Bad	3.5	1	4
Alternative 3	15	129	Very Good	2	4	1
Alternative 4	9	146	Very Bad	3.5	2	5
Alternative 5	7	121	Good	5	5	2.5

Aggregation methods

- Based on Ranks

3. Median rank
4. Majority
5. Borda’s Count

Median rank

3 units/alternatives: **A, B, C**

11 indicators/criteria:

5 indicators	**4** indicators	**2** indicators
A	C	C
C	A	B
B	B	A

Sort indicators by rank within each unit, calculate median rank for each unit, and rank units based on that median value

A: 11111**2**22233
B:22333**3**33333
C:11111**1**22222

Ranking
C
A
B

Aggregation methods

- Based on Ranks

3. Median rank
4. Majority (Relative Majority)
5. Borda's Count

Relative majority

Rank	Points
1	1
2	0
3	0
…	…
N-1	0
N	0

3 units/alternatives: **A, B, C**

30 indicators/criteria, **Only the first position counts**

13 indicators	**10** indicators	**7** indicators
A	B	C
B	C	B
C	A	A

In this example, **A** is the least preferred option in 17 indicators out of 30 (i.e. although **A** is ranked first, for a majority of indicators **B** and **C** are still preferred to **A**)

Problem: The alternative most often ranked first might also be the one with the strongest opposition (***Relative majority paradox***)

A	13
B	10
C	7

A is ranked first

Aggregation methods

- Based on Ranks

3. Median rank
4. Majority (Relative Majority)
5. Borda’s Count

Borda's count

Rank	Borda points
1	N-1
2	N-2
3	N-3
…	…
N-1	1
N	0

3 units/alternatives: **A, B, C**
81 indicators/criteria:

30 indicators	**29** indicators	**10** indicators	**10** indicators	**1** indicators	**1** indicators
A	C	C	B	A	B
C	A	B	A	B	C
B	B	A	C	C	A

Points
2
1
0

Scores	
A	101
B	33
C	109

C is ranked first:

C = (29 + 10) x 2 + (30 + 1) x 1 = 109

European Commission

Borda's count with unequal weights

Rank	Borda points
1	N-1
2	N-2
3	N-3
…	…
N-1	1
N	0

3 units/alternatives: **A, B, C**

6 indicators/ criteria **with (unequal) weights**

C is ranked first

C = (0.30 + 0.15) x 2 + (0.05 + 0.20) x 1= 1.15

Ind. 1	Ind. 2	Ind. 3	Ind. 4	Ind. 5	Ind. 6
0.05 weight	**0.30** weight	**0.15** weight	**0.10** weight	**0.10** weight	**0.20** weight
A	C	C	B	A	B
C	A	B	A	B	C
B	B	A	C	C	A

Points
2
1
0

(weighted) Scores	
A	0.70
B	0.85
C	1.15

Borda's count and the problem of irrelevant alternatives

Rank	Borda points
1	N-1
2	N-2
3	N-3
…	…
N-1	1
N	0

4 units/alternatives: **A, B, C, D**

7 indicators/criteria:

3 indicators	**2** indicators	**2** indicators	Points
C	B	A	3
B	A	D	2
A	D	C	1
D	C	B	0

Scores	
A	13
B	12
C	11
D	6

Ranking
A
B
C
D

A is ranked first

Borda's count and the problem of irrelevant alternatives

Rank	Borda points
1	N-1
2	N-2
3	N-3
…	…
N-1	1
N	0

Let's exclude now the least popular alternative:

3 units/alternatives: **A, B, C, ~~D~~**

7 indicators/criteria:

3 indicators	2 indicators	2 indicators	Points
C	B	A	2
B	A	C	1
A	C	B	0

In this example, excluding the least popular (seemingly irrelevant) alternative would imply reversing the ranks of all other alternatives

Problem: Borda's count is ***highly dependent on irrelevant alternatives (rank reversals occur often – high risk of manipulating results)***

Scores	
A	6
B	7
C	8

Ranking
C
B
A

C is first and the ranking has been reversed!

European Commission

Aggregation methods

- Based on Pairwise Comparisons

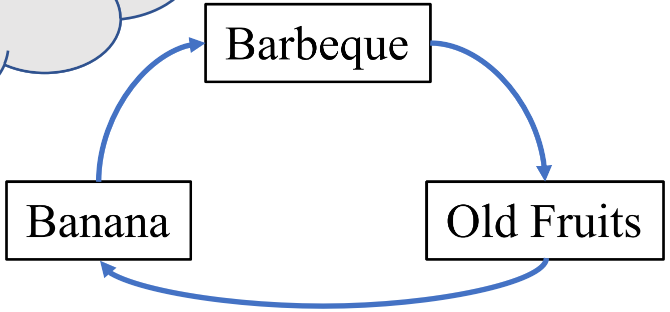
  6. Condorcet
  7. Kemeny
  8. Arrow – Raynaud
  9. Copeland

European Commission

Back to the forest – what *if* the monkey can't calculate?

Safety: Danger! Humans
Taste: I love meat!
Distance: Needs little walk

I would really enjoy a decision-making tool



Safety: Safe
Taste: Same old
Distance: Next to me

Safety: Can it be more safe?
Taste: Meh
Distance: Far

European Commission

Aggregation methods

- Based on Pairwise Comparisons

6. Condorcet
7. Kemeny
8. Arrow – Raynaud
9. Copeland

Condorcet's Method

	Ind 1	Ind 2	Ind 3	Ind 4	Ind 5
Country A	58.2	51.1	59.6	79.7	28.2
Country B	88.3	50.6	68.8	69.6	45.8
Country C	77.9	34.4	50.2	47.5	48.9

Preference	Country A	Country B	Country C
Country A	-	2	3
Country B	3	-	4
Country C	2	1	-

The **Condorcet winner** is the alternative preferred over every other in pairwise comparisons

Considering 5 Indicators:

Country B is the preferred alternative

Ranking: B > A > C

European Commission

Condorcet's Method: the problem with cycles

	Ind 1	Ind 2	Ind 3	Ind 4	Ind 5	Ind 6	Ind 7	Ind 8	Ind 9	Ind 10	Ind 11	Ind 12	Ind 13	Ind 14	Ind 15	Ind 16
Country A	58.2	51.1	59.6	79.7	28.2	65.5	56.0	77.1	15.4	76.4	51.9	96.5	97.1	51.9	60.9	50.5
Country B	88.3	50.6	68.8	69.6	45.8	63.6	75.8	71.3	28.2	68.2	82.6	87.7	97.0	49.8	58.7	81.2
Country C	77.9	34.4	50.2	47.5	48.9	46.3	77.4	65.7	15.0	58.0	56.6	97.1	98.7	47.6	67.0	51.1

Preference	Country A	Country B	Country C
Country A	-	9	7
Country B	7	-	11
Country C	9	5	-

Problem: Sometimes is not possible to find a Condorcet winner (**"circular ambiguity"**)

A > B, B > C and C > A

No alternative preferred to all others – a further method must be used to choose the winner

Aggregation methods

- Based on Pairwise Comparisons

6. Condorcet
7. Kemeny (C-K-Y-L)
8. Arrow – Raynaud
9. Copeland

European Commission

Importance of Indicators: The Outranking Matrix

	Ind 1	Ind 2	Ind 3	Ind 4	Ind 5
Country A	58	51	59	79	28
Country B	88	50	68	69	45
Country C	77	34	50	47	48
Weight	0.1	0.3	0.1	0.3	0.2

The outranking matrix collects the results of the pairwise comparisons

Preference	Country A	Country B	Country C
Country A	-	0.6	0.7
Country B	0.4	-	0.9
Country C	0.3	0.1	-

Concordance Values

Considering the weights:

Country A is the preferred alternative

Ranking: A > B >C

How to compute the concordance values

For every pairwise comparison we need to compute the *concordance value*

<u>Note</u>: weights can't be ≥ 50%; in case of ties, weights are split between the pair

Indicator	I.1	I.2	I.3
Weights	0.35	0.45	0.20
Unit A	3	Very Bad	205
Unit B	4	Bad	48
Unit C	3	Very Bad	88
Unit D	6	Very Good	446
Unit E	2	Good	208
Unit F	4	Bad	88
Unit G	3	Good	351
Unit H	5	Bad	88

Example 1:

A versus B = 0.20

B versus A = 0.35 + 0.45 = 0.80

Example 2:

D versus G = 0.35 + 0.45 + 0.20 = 1.00

G versus D = 0.00

Example 3:

F versus H = 0.225 + 0.10 = 0.325

H versus F = 0.35 + 0.225 + 0.10 = 0.675

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Outranking Matrix – Construction

Data 2013	Fiscal Dimension			
	2.5	2.6	2.7	2.8
Orientation	-1	-1	-1	-1
Weights	0.25	0.25	0.25	0.25
MT	74.9	3.6	205.0	14.7
NL	75.8	3.5	47.9	11.6
PL	58.9	4.1	87.9	5.8
PT	124.3	4.0	445.5	18.8
RO	38.5	2.4	207.6	7.3
SE	39.0	0.4	17.7	7.0
SI	66.5	4.9	350.9	5.6
SK	56.7	3.1	87.8	9.2

Step 1 – Raw data, Weights & Orientation

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Outranking Matrix – Construction

Data 2013	Fiscal Dimension			
	2.5	2.6	2.7	2.8
Orientation	-1	-1	-1	-1
Weights	0.25	0.25	0.25	0.25
MT	74.9	3.6	205.0	14.7
NL	75.8	3.5	47.9	11.6
PL	58.9	4.1	87.9	5.8
PT	124.3	4.0	445.5	18.8
RO	38.5	2.4	207.6	7.3
SE	39.0	0.4	17.7	7.0
SI	66.5	4.9	350.9	5.6
SK	56.7	3.1	87.8	9.2

Step 1 – Raw data & Weights & Orientation

Step 2 – Concordance value

For *n* countries, there are *n (n-1)* pairwise comparisons to be made

Example:
MT versus NL = 0.25
NL versus MT = 0.75

Sum = 1.00

European Commission

Outranking Matrix – Construction

Data 2013	Fiscal Dimension			
	2.5	2.6	2.7	2.8
Orientation	-1	-1	-1	-1
Weights	0.25	0.25	0.25	0.25
MT	74.9	3.6	205.0	14.7
NL	75.8	3.5	47.9	11.6
PL	58.9	4.1	87.9	5.8
PT	124.3	4.0	445.5	18.8
RO	38.5	2.4	207.6	7.3
SE	39.0	0.4	17.7	7.0
SI	66.5	4.9	350.9	5.6
SK	56.7	3.1	87.8	9.2

Step 1 – Raw data & Weights & Orientation

Step 2 – Concordance value

For *n* countries, there are ***n (n-1)* pairwise comparisons** to be made

Example:
MT versus PT = 1.00
PT versus MT = 0.00

Sum = 1.00 → **Robust pair**

Outranking Matrix – Construction

Step 1 – Raw data & Weights & Orientation

Step 2 – Concordance value

Step 3 – **Outranking matrix**

	MT	NL	PL	PT	RO	SE	SI	SK
MT	0.00	0.25	0.25	1.00	0.25	0.00	0.50	0.00
NL	0.75	0.00	0.50	1.00	0.25	0.00	0.50	0.25
PL	0.75	0.50	0.00	0.75	0.50	0.25	0.75	0.50
PT	0.00	0.00	0.25	0.00	0.00	0.00	0.25	0.00
RO	0.75	0.75	0.50	1.00	0.00	0.25	0.75	0.75
SE	1.00	1.00	0.75	1.00	0.75	0.00	0.75	1.00
SI	0.50	0.50	0.25	0.75	0.25	0.25	0.00	0.25
SK	1.00	0.75	0.50	1.00	0.25	0.00	0.75	0.00

All concordance values are entered in the outranking matrix.
(entries above and below the diagonal sum up to 1.0)

MT versus NL = 0.25
NL versus MT = 0.75

MT versus PT = 1.00
PT versus MT = 0.00

Kemeny order *Condorcet-Kemeny-Young-Levenglick (C-K-Y-L)*

	MT	NL	PL	PT	RO	SE	SI	SK
MT	0.00	0.25	0.25	**1.00**	0.25	0.00	0.50	0.00
NL	**0.75**	0.00	0.50	**1.00**	0.25	0.00	0.50	0.25
PL	**0.75**	0.50	0.00	**0.75**	0.50	0.25	**0.75**	0.50
PT	0.00	0.00	0.25	0.00	0.00	0.00	0.25	0.00
RO	**0.75**	**0.75**	0.50	**1.00**	0.00	0.25	**0.75**	**0.75**
SE	**1.00**	**1.00**	**0.75**	**1.00**	**0.75**	0.00	**0.75**	**1.00**
SI	0.50	0.50	0.25	**0.75**	0.25	0.25	0.00	0.25
SK	**1.00**	**0.75**	0.50	**1.00**	0.25	0.00	**0.75**	0.00

Step 1 – Raw data & Weights & Orientation

Step 2 – Concordance value

Step 3 – Outranking matrix

Step 4 – Maximum Likelihood ranking (highest support score)

Find the **permutation of rankings** which **maximises the sum** of elements **above the diagonal**

Kemeny order – Maximises Likelihood

	SE	RO	PL	SK	SI	NL	MT	PT
SE	0.00	**0.75**	**0.75**	**1.00**	**0.75**	**1.00**	**1.00**	**1.00**
RO	0.25	0.00	0.50	**0.75**	**0.75**	**0.75**	**0.75**	**1.00**
PL	0.25	0.50	0.00	0.50	**0.75**	0.50	**0.75**	**0.75**
SK	0.00	0.25	0.50	0.00	**0.75**	**0.75**	**1.00**	**1.00**
SI	0.25	0.25	0.25	0.25	0.00	0.50	0.50	**0.75**
NL	0.00	0.25	0.50	0.25	0.50	0.00	**0.75**	**1.00**
MT	0.00	0.25	0.25	0.00	0.50	0.25	0.00	**1.00**
PT	0.00	0.00	0.25	0.00	0.25	0.00	0.00	0.00

Step 1 – Raw data & Weights & Orientation

Step 2 – Concordance value

Step 3 – Outranking matrix

Step 4 – Maximum Likelihood ranking (highest support score)

Find the **permutation of rankings** which **maximises the sum** of elements **above the diagonal**

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Kemeny order – Maximises Likelihood

	SE	RO	PL	SK	SI	NL	MT	PT
SE	0.00	**0.75**	**0.75**	**1.00**	**0.75**	**1.00**	**1.00**	**1.00**
RO	0.25	0.00	0.50	**0.75**	**0.75**	**0.75**	**0.75**	**1.00**
PL	0.25	0.50	0.00	0.50	**0.75**	0.50	**0.75**	**0.75**
SK	0.00	0.25	0.50	0.00	**0.75**	**0.75**	**1.00**	**1.00**
SI	0.25	0.25	0.25	0.25	0.00	0.50	0.50	**0.75**
NL	0.00	0.25	0.50	0.25	0.50	0.00	**0.75**	**1.00**
MT	0.00	0.25	0.25	0.00	0.50	0.25	0.00	**1.00**
PT	0.00	0.00	0.25	0.00	0.25	0.00	0.00	0.00

Step 1 – Raw data & Weights & Orientation

Step 2 – Concordance value

Step 3 – Outranking matrix

Step 4 – Maximum Likelihood ranking (highest support score)

	Rank
SE	1
RO	2
PL	3
SK	4
SI	5
NL	6
MT	7
PT	8

A Kemeny order is not always unique!
(there might be several rankings with the same maximum support score)

Aggregation methods

- Based on Pairwise Comparisons

6. Condorcet
7. Kemeny (C-K-Y-L)
8. Arrow – Raynaud
9. Copeland

Quick-searching algorithms have been developed to approximate the optimal solution of Kemeny order

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Arrow-Raynaud algorithm

- Arrow-Raynaud algorithm selects rankings that resolve cycles (based on a minimization function)
- The number of overall rankings identified as potential solutions to the cycle can be LARGE.

11 Equivalent rankings resulting from our example

MT	**7**	7	7	7	7	7	7	6	6	6	6
NL	**6**	6	6	5	5	5	4	5	5	4	5
PL	**3**	4	2	4	3	2	5	4	3	5	2
PT	**8**	8	8	8	8	8	8	8	8	8	8
RO	**2**	2	3	2	2	3	2	2	2	2	3
SE	**1**	1	1	1	1	1	1	1	1	1	1
SI	**5**	5	5	6	6	6	6	7	7	7	7
SK	**4**	3	4	3	4	4	3	3	4	3	4

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Aggregation methods

- Based on Pairwise Comparisons

  6. Condorcet
  7. Kemeny (C-K-Y-L)
  8. Arrow – Raynaud
  9. Copeland

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Solution to the dilemma…

Safety: Bad
Taste: Very Good
Distance: Average

Safety: Good
Taste: Average
Distance: Very Good

Safety: Very Good
Taste: Bad
Distance: Bad

How does Copeland work? Counts ***Victories (+1)*** and ***Defeats (-1)*** (***ties don't count***)

Outranking matrix

Preference	Barbeque	Banana	Old Fruits
Barbeque	-	1	2
Banana	2	-	2
Old Fruits	1	1	-

	Wins	Defeats	Score	Rank
Banana	2	0	2	1
Barbeque	1	1	0	2
Old Fruits	0	2	-2	3

A Realistic example

Victories (+1) minus ***Defeats (-1)*** (***ties don't count***)

Outranking matrix

	MT	NL	PL	PT	RO	SE	SI	SK
MT	0.00	0.25	0.25	1.00	0.25	0.00	0.50	0.00
NL	0.75	0.00	0.50	1.00	0.25	0.00	0.50	0.25
PL	0.75	0.50	0.00	0.75	0.50	0.25	0.75	0.50
PT	0.00	0.00	0.25	0.00	0.00	0.00	0.25	0.00
RO	0.75	0.75	0.50	1.00	0.00	0.25	0.75	0.75
SE	1.00	1.00	0.75	1.00	0.75	0.00	0.75	1.00
SI	0.50	0.50	0.25	0.75	0.25	0.25	0.00	0.25
SK	1.00	0.75	0.50	1.00	0.25	0.00	0.75	0.00

	Wins	Defeats	Total	Rank
SE	7	0	7	1
RO				
PL				
SK				
SI				
NL				
MT				
PT				

Copeland rule

If Kemeny is the optimal solution, Copeland is a good approximation with the advantage of "button-click" speed

Outranking matrix

	MT	NL	PL	PT	RO	SE	SI	SK
MT	0.00	0.25	0.25	1.00	0.25	0.00	0.50	0.00
NL	0.75	0.00	0.50	1.00	0.25	0.00	0.50	0.25
PL	0.75	0.50	0.00	0.75	0.50	0.25	0.75	0.50
PT	0.00	0.00	0.25	0.00	0.00	0.00	0.25	0.00
RO	0.75	0.75	0.50	1.00	0.00	0.25	0.75	0.75
SE	1.00	1.00	0.75	1.00	0.75	0.00	0.75	1.00
SI	0.50	0.50	0.25	0.75	0.25	0.25	0.00	0.25
SK	1.00	0.75	0.50	1.00	0.25	0.00	0.75	0.00

	Wins	Defeats	Total	Rank
SE	7	0	7	1
RO	5	1	4	2
SK	4	2	2	3
PL	3	1	2	4
NL	2	3	-1	5
SI	1	4	-3	6
MT	1	5	-4	7
PT	0	6	-6	8

It is in the COIN tool (ask us for R and Matlab code)

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Summary of methods on Pairwise Comparison

- Fully ***non-compensatory*** approach;
- ***only weights and orientation*** are ***required*** to obtain the ranking of alternatives;
- ***weights*** represent exactly ***the importance*** *of the indicator*;
- no impact of outliers;
- no need for data normalisation
- no need for ”good” correlation structure;
- can be used both with continuous and categorical variables;
- computationally more demanding than standard averages;
- poor with small number of units;
- software available for Copeland (send an email to JRC-COIN): Excel, R, Matlab

Sources: Athanasoglou (2015) , Tarjan (1972), Van Zuylen, and Williamson (2009), Munda and Nardo (2009)

Super-small last example

	Math	Science	History
Albert	10	8	9
Howard	6	6	6
John	3	4	10
Amy	2	10	6

Arithmetic	Geometric	Borda	Copeland
9.00	8.96	7	3
6.00	6.00	3.5	0
5.67	4.93	4	-1
6.00	4.93	3.5	-2

Correlation matrix			
	Math	Science	History
Math	1	0.036	0.236
Science	0.036	1	-0.564
History	0.236	-0.564	1

Revisiting aggregation approaches

- Based on Average Scores
  1. Arithmetic Mean
  2. Geometric Mean

"Absolute" approaches: each unit's performance is almost independent of other units / output: scores

- Based on Ranks
  3. Median rank
  4. Majority (Relative Majority)
  5. Borda's Count

"Compensatory" approaches (weights interpreted as trade-offs)

- Based on Pairwise Comparisons
  6. Condorcet
  7. Kemeny (C-K-Y-L)
  8. Arrow – Raynaud
  9. Copeland

"Non-Compensatory" approaches (weights interpreted as importance coefficients)

"Relative" approaches: each unit's performance depends on other unit's performance: ranks (*part of numerical information is lost*)
- no normalisation needed
- not influenced by outliers

THANK YOU

Welcome to email us at: jrc-coin@ec.europa.eu

COIN in the EU Science Hub
https://ec.europa.eu/jrc/en/coin

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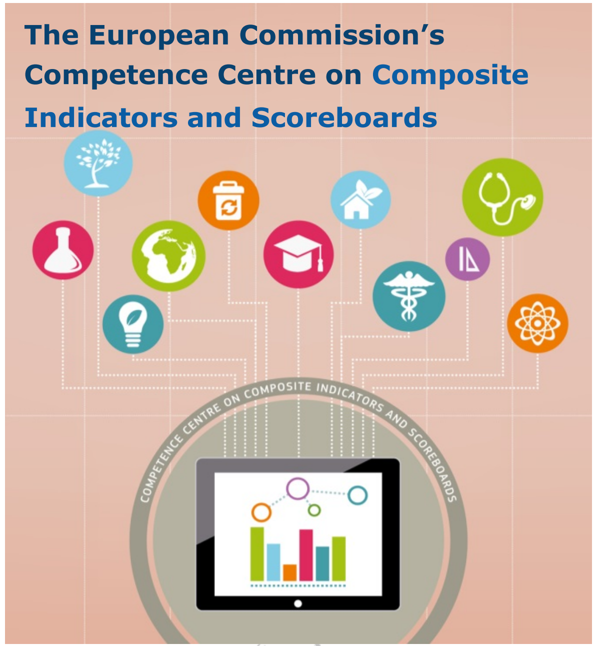

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Suggested readings

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