The European Commission's science and knowledge service

Joint Research Centre

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Step 5: Weighting methods
Budget allocation, Analytic Hierarchy Process

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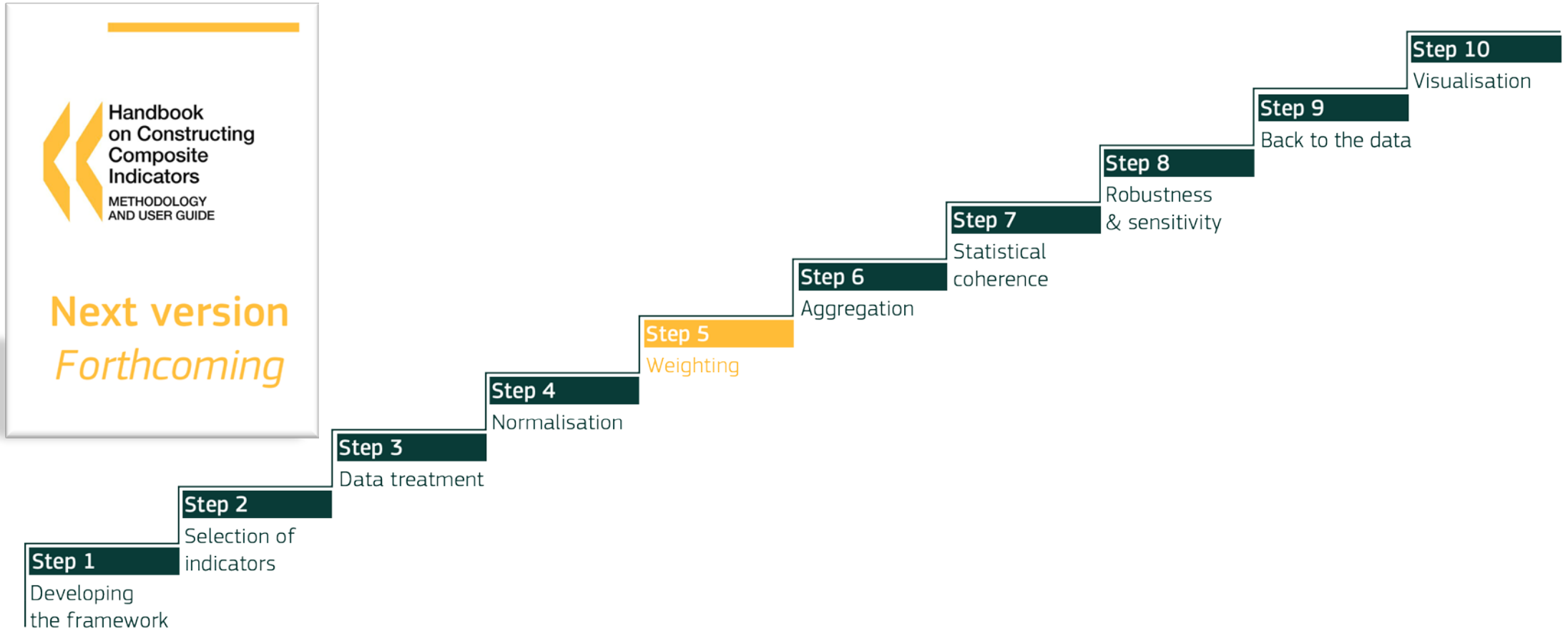
Ten steps



Weights

- Equal weights

Weights based on statistical models

- Principal component/Factor analysis
- Data envelopment analysis
- Regression approaches

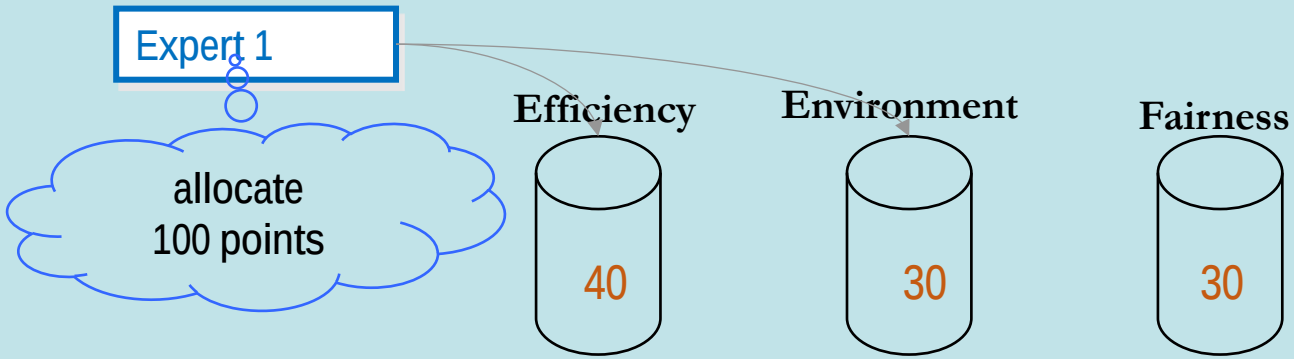
Weights based on participation

- *Budget allocation*
- *Analytic hierarchy process*
- Conjoint analysis

Budget Allocation - BAL

Phases

1. Selection of **experts/stakeholders** for the evaluation;
   a. Number
   b. Background/Expertise
2. **Allocation** of budget to indicators;



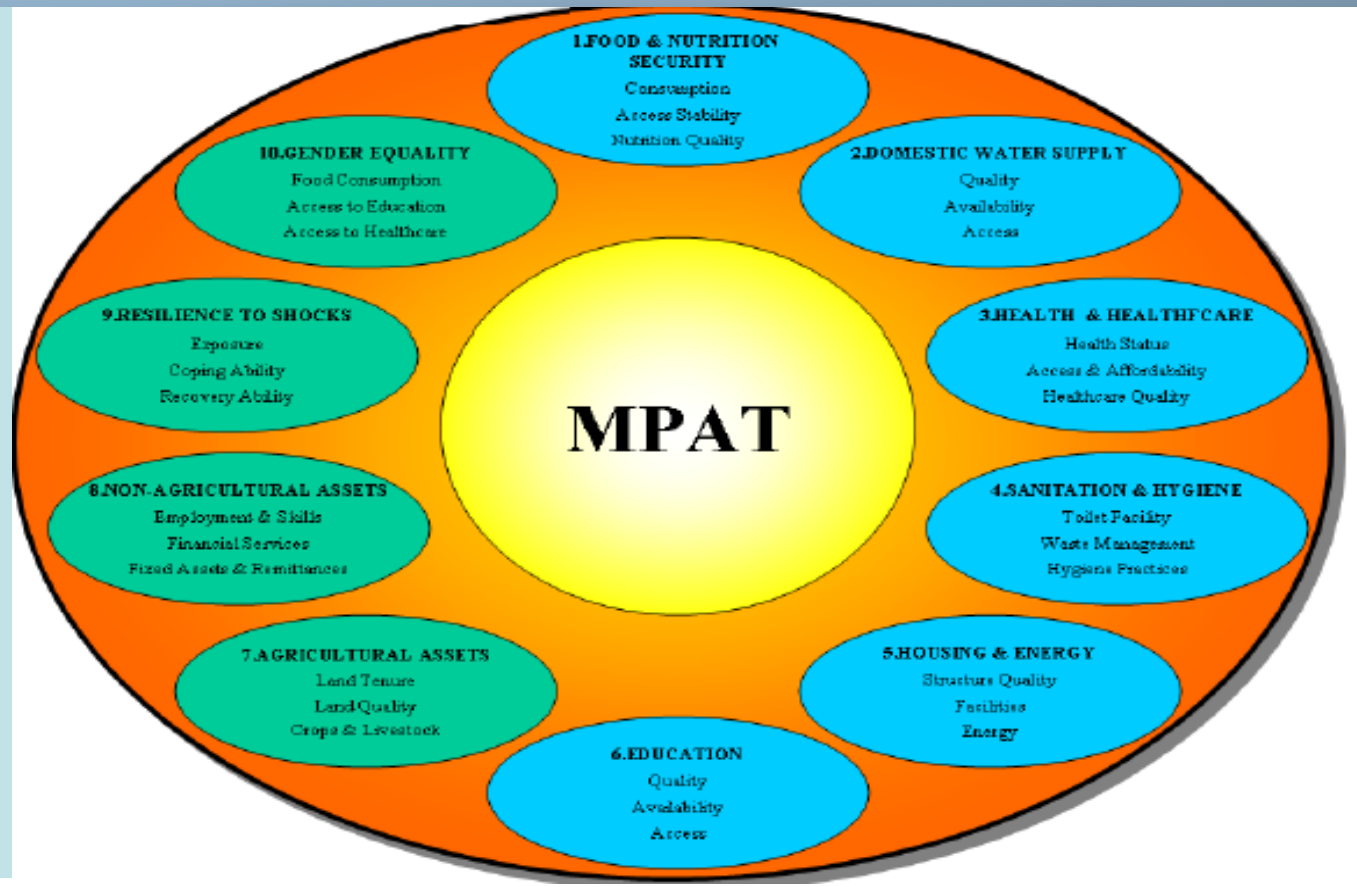
Suited for up to **8-10** indicators

3. Calculation of **weights**;
4. Iteration of the budget allocation until convergence is reached (optional)

MPAT - snapshot

Source: *Quantifying the qualitative: Eliciting expert input to develop the Multidimensional Poverty Assessment Tool* (Cohen, Saisana, J of Dev. Studies, 2014, 50(1))

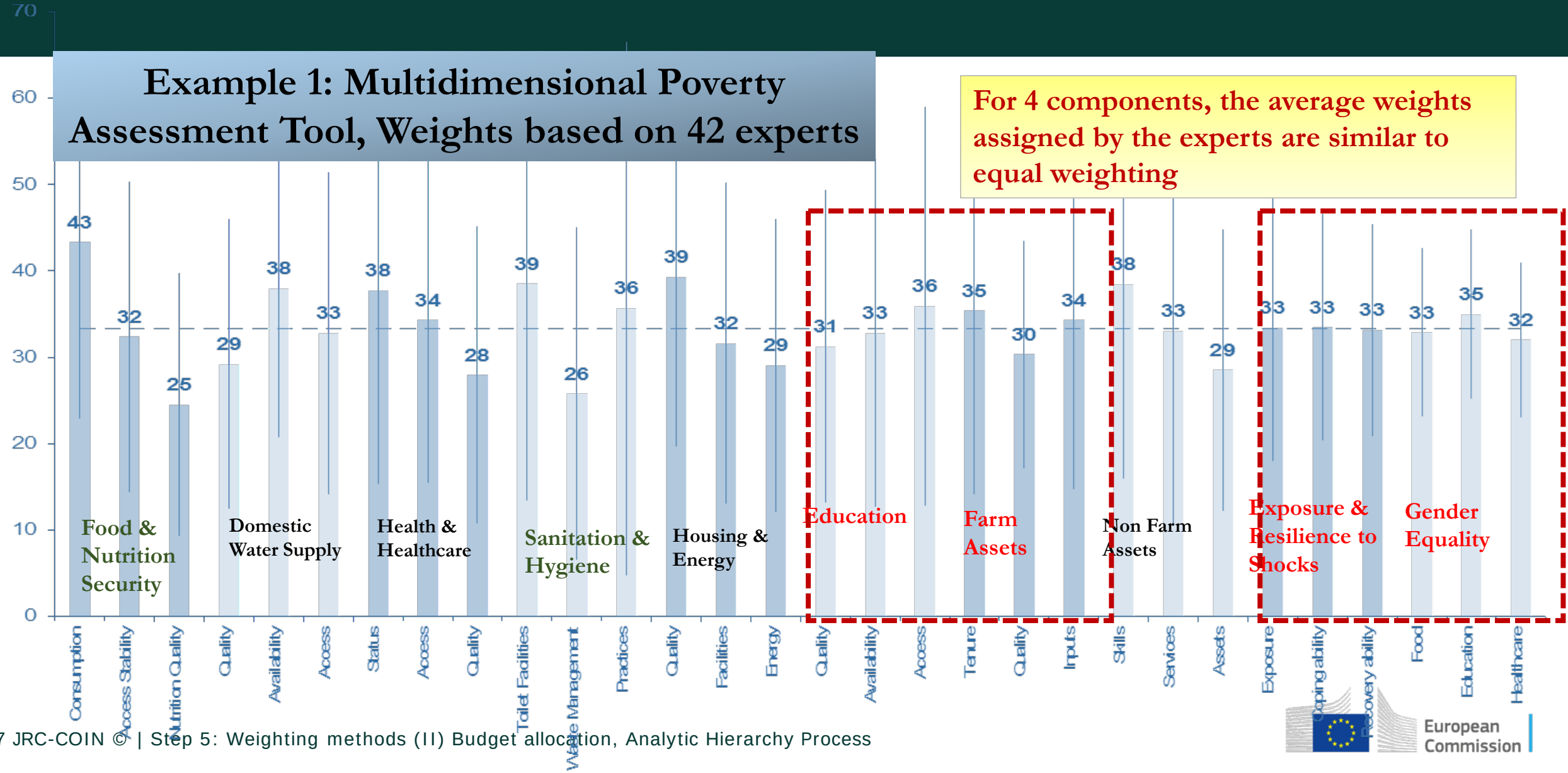
Example 1: Multidimensional Poverty Assessment Tool, Weights based on 42 experts



- **BAL**
  - **Purpose**: Eliciting weights to be assigned to the **subcomponents of each of the dimensions**
- **Selection of Experts**
  - 42 Experts from 10 countries and 28 organizations
  - Mainly from UN agencies and universities
  - Selection based on expertise on poverty assessme tools in developing countries

- No real sampling frame

MPAT - Results of the Budget Allocation



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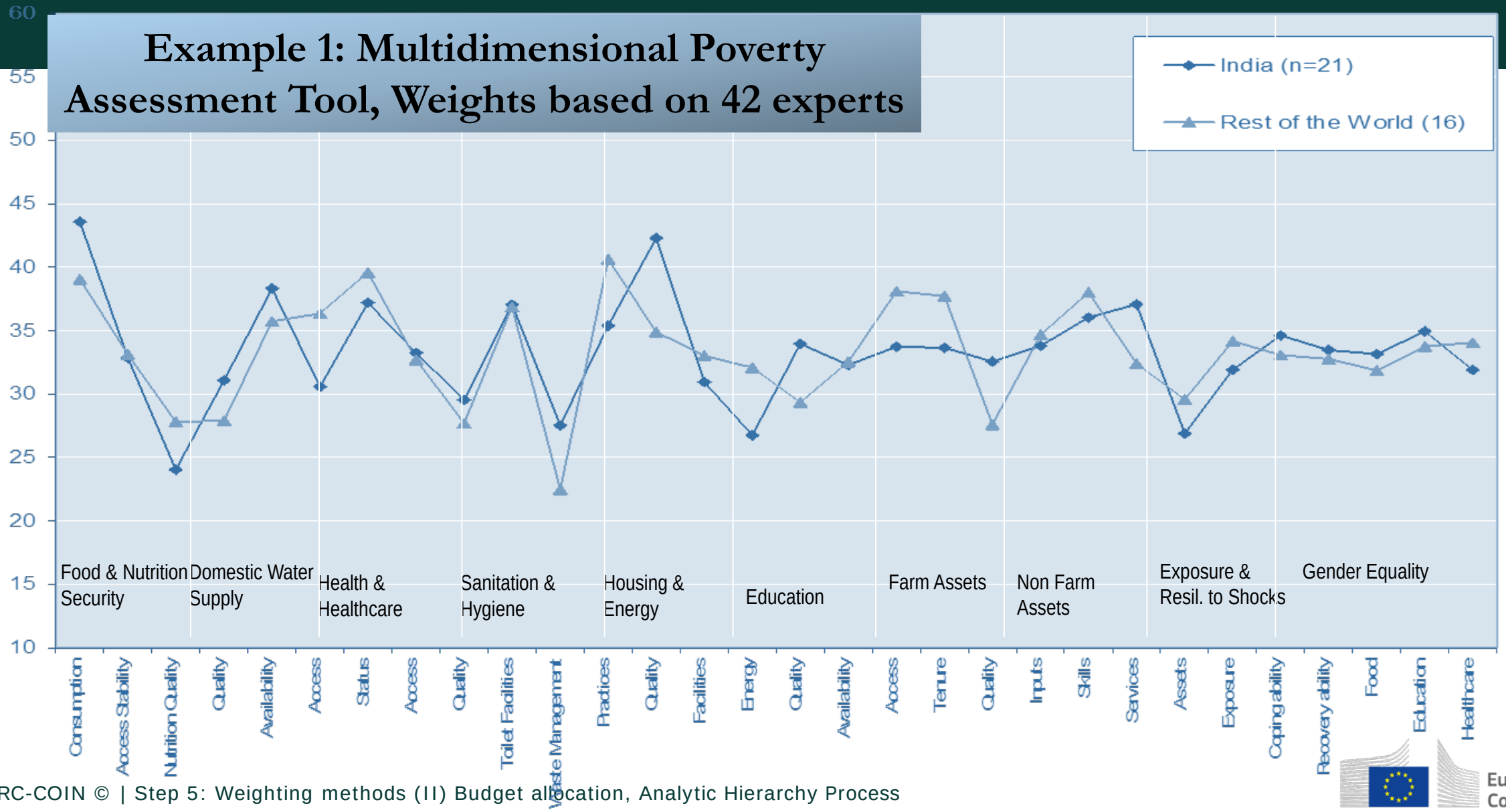
MPAT - Results of the Budget Allocation

Example 1: Multidimensional Poverty Assessment Tool, Weights based on 42 experts



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MPAT - Results of the Budget Allocation

Example 1: Multidimensional Poverty Assessment Tool, Weights based on 42 experts

For 10 subcomponents China experts have very different views comparatively to the experts coming from India or the Rest of the world

C3 Index - Snapshot

Example 2 : The Cultural and Creative Cities Monitor, 2019 Edition

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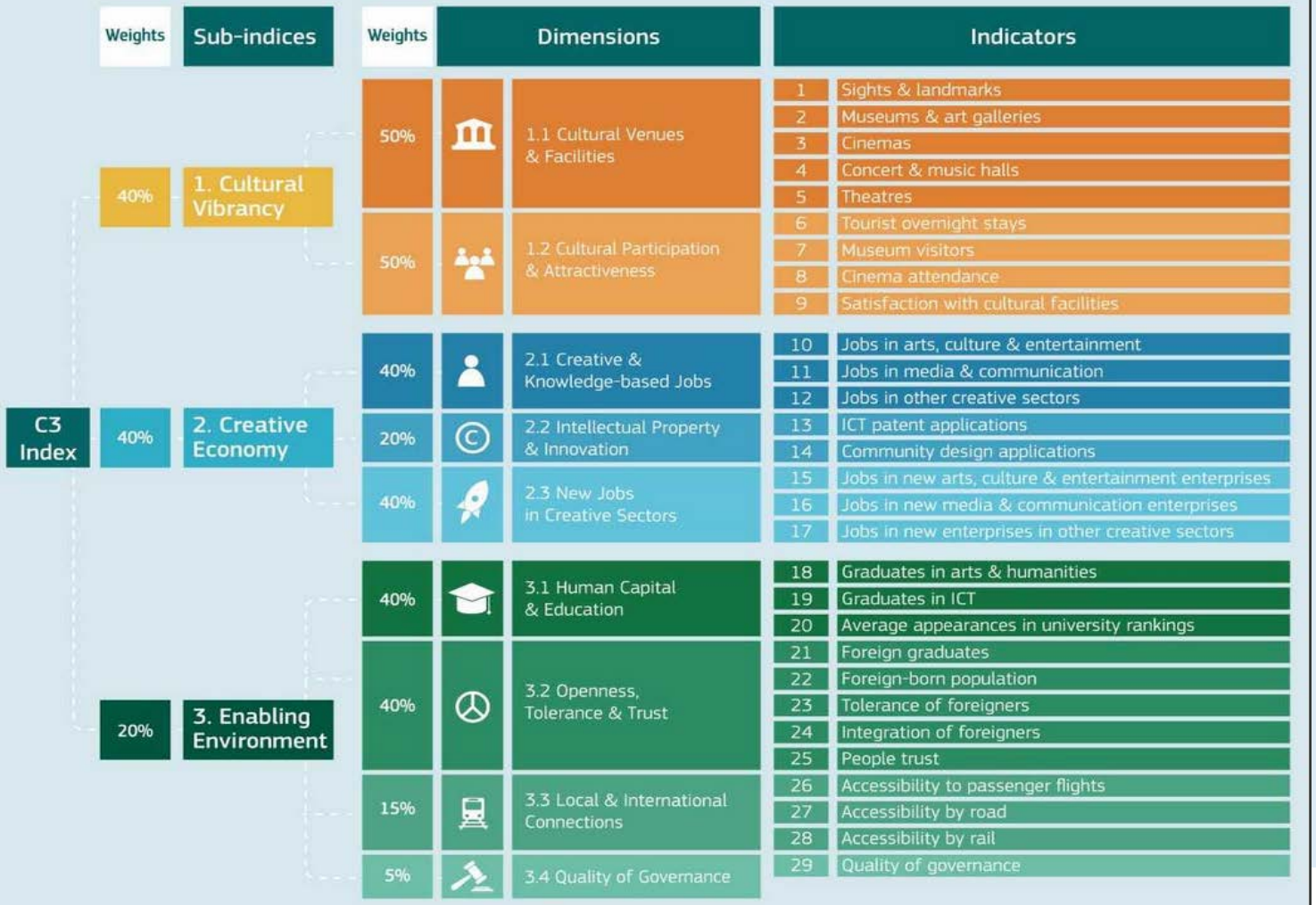
C3 Index - Snapshot

Example 2 : The Cultural and Creative Cities Monitor, 2019 Edition



- **BAL** : Eliciting weights to be assigned to
  - the **3 sub-indices**
  - the **9 dimensions**

- **Selection of Experts**
  - **17 Experts**
    - 5 from EC, 6 from Academia, 6 from international organisations
  - Experts divided in **3 groups**
- **When**
  - Second participatory workshop of the C3 Monitor - November 2016

C3 Index - BAL

Example 2 : The Cultural and Creative Cities Monitor, 2019 Edition

	Group 3	Group 2	Group 1	Average of the 3 groups	Final Weight
Cultural Vibrancy	40	50	40	43.3	40
Creative Economy	40	30	35	35.0	40
Enabling Environment	20	20	25	21.7	20

Weights assigned to the **three sub-indices** by each group

«Enabling Environment» sub-indice

- Emerged from the discussion that **accessibility** and **governance** dimensions should have a minimum weight

Human Capital & Education - Academic Appeal	**40**
Openness, Tolerance and Trust	40
Accessibility - local & international	15
Governance & Regulations	5

Suggestions for the BAL

- When possible- use a **sampling frame** to select the experts & **maximize response rate**

→*Compensating* experts might increase participation (Chowdury and Squire, 2006)

- **Experts** with balance of **diverse backgrounds**
- Collect information on the **characteristics** of the experts (Cooke, 1991)
- During the survey, **do not bother about the "100 points" sum** when there are more than 4 indicators (rescale to 100 after the survey).
- **Randomize** the order of the components, so that some experts evaluate first component A and others component B, and so on.

Analytic Hierarchy Process

- **Multi-criteria decision making method**
- Developed by **Thomas Saaty (1980, 1987)**

Recommanded for **less than 10 indicators**

Phases

1. Selection of **experts/stakeholders** for the evaluation;
2. **Pairwise comparisons** of indicators on a scale 1 to 9 (1: equally important, 9: most important);
3. Calculation of **weights** through the derivation of the priority vector;
4. Estimation of **consistency of the experts' assessment**.

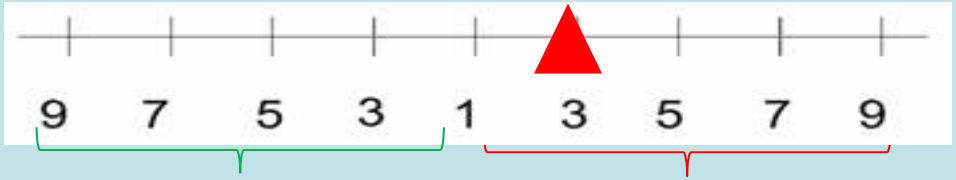
Analytic Hierarchy Process

Phase 2- PAIRWISE COMPARISONS to express THE **RELATIVE** IMPORTANCE OF ONE INDICATOR OVER ANOTHER

Which indicator do you feel is more important?

1 EQUAL 3 MODERATE 5 STRONG 7 VERY STRONG 9 EXTREME

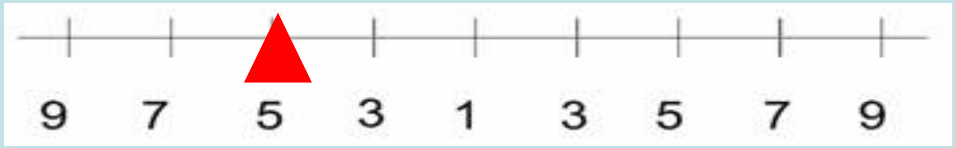
Indicator 1



Indicator 2

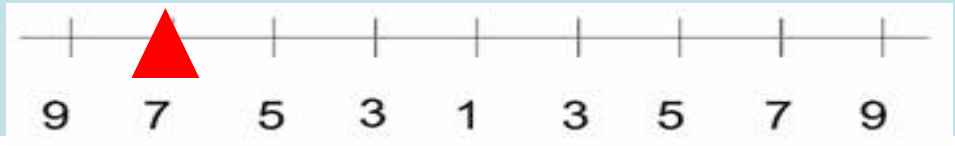
More time consuming than budget allocation : $n \cdot (n-1)/2$ comparisons needed

Indicator 1



Indicator 3

Indicator 2



Indicator 3

Analytic Hierarchy Process

Phase 2- PAIRWISE COMPARISONS to express THE **RELATIVE** IMPORTANCE OF ONE INDICATOR OVER ANOTHER

1 EQUAL 3 MODERATE 5 STRONG 7 VERY STRONG 9 EXTREME

- Set up a ***n* *n matrix (A)** with *n* being the number of indicators

Matrix A	Indicator 1	Indicator 2	Indicator 3
Indicator1	1	1/3	5
Indicator 2	3	1	7
Indicator 3	1/5	1/7	1

I_2 three times more important than I_1 I_{21}=3

I_{12} Reciprocal value of I_{21}

Analytic Hierarchy Process

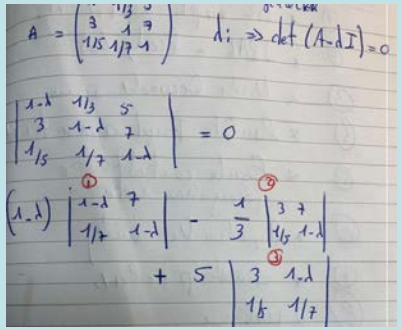
Phase 3- Calculation of **WEIGHTS**

For each matrix A , need to derive the **weights**
→ **different** methods

λ is called an eigenvalue if there is a nonzero vector x such that $Ax = \lambda x$. X is called an eigenvector of A corresponding to λ

Saaty (1990) shows that the weight vector is the **eigenvector of the matrix A** corresponding to the **highest eigenvalue**

```
mkmat indicator1 indicator2 indicator3, matrix(A)
matrix list A
matrix symeigen Eigenvector Eigenvalue=A
matrix list Eigenvector
matrix list Eigenvalue
```



Analytic Hierarchy Process

Phase 3- Proxy of the **weights vector** when the number of Indicators is limited –**normalized columns method**

a - **Sum** each column of the matrix

	I1	I 2	I3
I1	1	1/3	5
I2	3	1	7
I3	1/5	1/7	1
Sum	**21/5**	**31/21**	**13**

b - **Normalized** relative weights

	I1	I 2	I3
I1	5/21	7/31	5/13
I2	15/21	21/31	7/13
I3	1/21	3/31	1/13
Sum	1	1	1

c - **Average across the rows**

I.1	**0.2828**
I2	**0.6434**
I3	**0.0738**

Weights

Analytic Hierarchy Process

Phase 3- Proxy of the **weights vector** when the number of Indicators is limited –**normalized columns method**

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I3	1/5	1/7	1
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	I1	I 2	I3
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I2	15/21	21/31	7/13
I3	1/21	3/31	1/13
Sum	1	1	1

c - **Average across the rows**

I.1	**0.2828**
I2	**0.6434**
I3	**0.0738**

Weights

Analytic Hierarchy Process

Phase 3- Other method to retrieve the **weights vector** – **geometric mean method**

	I1	I 2	I3	geometric mean	normalized geometric mean
I1	1.00	0.33	5.00	1.19	0.28
I2	3.00	1.00	7.00	2.76	0.65
I3	0.20	0.14	1.00	0.31	0.07
Sum				4.25	

Weights

Analytic Hierarchy Process

Phase 4 - Estimation of **consistency** ratio

- Experts' assessment: are they consistent?

  Consistency: $I_{13} = I_{12} * I_{23}$

- Experts' assessment = **subjective** preferences
  - Some **inconsistencies** are acceptable

	I1	I 2	I3
I1	1	1/3	5
I2	3	1	7
I3	1/5	1/7	1

- For each expert: necessary to compute a **consistency ratio**

$$CR = \frac{\text{consistency index of matrix A}}{\text{consistency index of a random-like matrix}} = \frac{CI(A)}{CI(R)}$$

$$CI(A) = \frac{\lambda_{max} - n}{n-1}, \quad \lambda_{max} = n \text{ if } A \text{ consistent}$$

Suggested rule-of-thumb is **CR ≤0.1** although **0.2** is often cited – do not drastically affect the weights (Saaty, 1980).

TAI - Snapshot

Source: Saisana, Saltelli, 2008, *Expert Panel Opinion and Global Sensitivity Analysis for Composite Indicators*, Lecture Notes in Computational Science and Engineering 62, pp. 251-275.

Example 1 : Technological Achievement Index

Questionnaire

Which Indicator Do You Feel Is More Important?				To What Degree?										
				1	2	3	4	5	6	7	8	9		
	Patents	vs.	x	Royalties			x							
x	Patents	vs.		Internet				x						
x	Patents	vs.		Technology exports				x						
x	Patents	vs.		Telephones			x							
x	Patents	vs.		Electricity									x	
	Patents	vs.	x	Schooling years						x				
	Patents	vs.	x	University Students								x		
x	Royalties	vs.		Internet			x							
	Royalties	vs.	x	Technology exports				x						
x	Royalties	vs.		Telephones					x					
x	Royalties	vs.		Electricity									x	
	Royalties	vs.	x	Schooling years			x							
	Royalties	vs.	x	University Students				x						
	Internet	vs.	x	Technology exports						x				
x	Internet	vs.		Telephones		x								
x	Internet	vs.		Electricity		x								
	Internet	vs.	x	Schooling years							x			
	Internet	vs.	x	University Students						x				
x	Technology exports	vs.		Telephones					x					
x	Technology exports	vs.		Electricity									x	
	Technology exports	vs.	x	Schooling years				x						
	Technology exports	vs.	x	University Students					x					
x	Telephones	vs.		Electricity							x			
	Telephones	vs.	x	Schooling years									x	
	Telephones	vs.	x	University Students									x	
	Electricity	vs.	x	Schooling years									x	
	Electricity	vs.	x	University Students									x	
x	Schooling years	vs.		University Students		x								

- Measure how a country is **creating and diffusing new & existent technologies** and building a human skill base with **8 achievement indicators**
- **Original CI**: equal weight
- Departure from the original weithing scheme using an **AHP based on a survey of 20 scientists of the JRC**

TAI – Reciprocal matrix A of 1 expert

Example 1 : Technological Achievement Index

1 EQUAL 3 MODERATE 5 STRONG 7 VERY STRONG 9 EXTREME

USING PAIRWISE COMPARISONS to express THE RELATIVE IMPORTANCE OF ONE CRITERION OVER ANOTHER

	Patents	*Royalties*	*Internet*	*Tech.Exports*	*Telephones*	*Electricity*	*Schooling*	*University St.*
Patents	1	1/3	5	4	3	9	1/6	1/8
Royalties	3	1	3	1/4	5	9	1/3	1/4
Internet	1/5	1/3	1	1/6	2	2	1/7	1/6
Tech.Exports	1/4	4	6	1	5	9	1/4	1/5
Telephones	1/3	1/5	1/2	1/5	1	7	1/9	1/9
Electricity	1/9	1/9	1/2	1/9	1/7	1	1/9	1/9
Schooling	6	3	7	4	9	9	1	2
University St.	8	4	6	5	9	9	1/2	1

TAI –Reciprocal matrix A- any inconsistency?

Example 1 : Technological Achievement Index

	Patents	Royalties	Internet	Tech.Exports	Telephones	Electricity	Schooling	University St.
Patents	1	1/3	5	4	3	9	1/6	1/8
Royalties	3	1	3	1/4			1/3	1/4
Internet	1/5	1/3	1	1/6			1/7	1/6
Tech.Exports	1/4	4	6	1			1/4	1/5
Telephones	1/3	1/5	1/2	1/5	1	7	1/9	1/9
Electricity	1/9	1/9	1/2	1/9	1/7	1	1/9	1/9
Schooling	6	3	7	4	9	9	1	2
University St.	8	4	6	5	9	9	1/2	1

Try to spot the inconsistency…

For a matrix of size Q × Q, only Q–1 comparisons are required to establish weights for Q indicators. But the number of AHP comparisons is Q(Q–1)/2.

TAI – Results of the AHP

Example 1 : Technological Achievement Index

Questionnaire

Which Indicator Do You Feel Is More Important? To What Degree?

				1	2	3	4	5	6	7	8	9
	Patents	vs.	x Royalties			x						
x	Patents	vs.	Internet					x				
x	Patents	vs.	Technology exports				x					
x	Patents	vs.	Telephones			x						
x	Patents	vs.	Electricity									x
	Patents	vs.	x Schooling years						x			
	Patents	vs.	x University Students								x	
x	Royalties	vs.	Internet			x						
	Royalties	vs.	x Technology exports				x					
x	Royalties	vs.	Telephones					x				
x	Royalties	vs.	Electricity									x
	Royalties	vs.	x Schooling years			x						
	Royalties	vs.	x University Students				x					
	Internet	vs.	x Technology exports						x			
x	Internet	vs.	Telephones		x							
x	Internet	vs.	Electricity		x							
	Internet	vs.	x Schooling years							x		
	Internet	vs.	x University Students						x			
x	Technology exports	vs.	Telephones					x				
x	Technology exports	vs.	Electricity									x
	Technology exports	vs.	x Schooling years				x					
	Technology exports	vs.	x University Students					x				
x	Telephones	vs.	Electricity							x		
	Telephones	vs.	x Schooling years									x
	Telephones	vs.	x University Students									x
	Electricity	vs.	x Schooling years									x
	Electricity	vs.	x University Students									x
x	Schooling years	vs.	University Students		x							

	Patents	Royalties	Internet	Tech.Exports	Telephones	Electricity	Schooling	University St.
Patents	1	1/3	5	4	3	9	1/6	1/8
Royalties	3	1	3	1/4	5	9	1/3	1/4
Internet	1/5	1/3	1	1/6	2	2	1/7	1/6
Tech.Exports	1/4	4	6	1	5	9	1/4	1/5
Telephones	1/3	1/5	1/2	1/5	1	7	1/9	1/9
Electricity	1/9	1/9	1/2	1/9	1/7	1	1/9	1/9
Schooling	6	3	7	4	9	9	1	2
University St.	8	4	6	5	9	9	1/2	1

solve for the Eigenvector

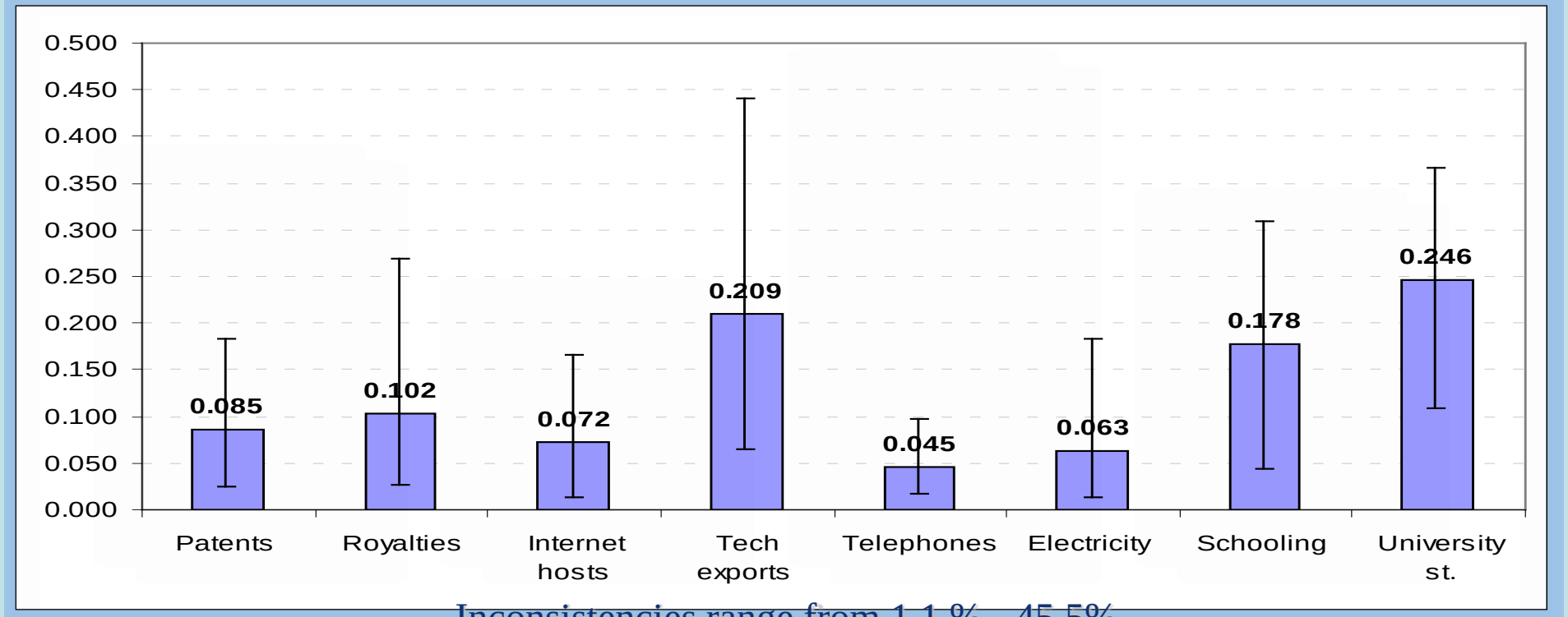
Weights

Patents	0.109
Royalties	0.103
Internet hosts	0.029
Tech exports	0.117
Telephones	0.030
Electricity	0.014
Schooling	0.301
University st.	0.297

Inconsistency
17.4 %

TAI – Result of the AHP - 18 weights vectors

Example 1 : Technological Achievement Index

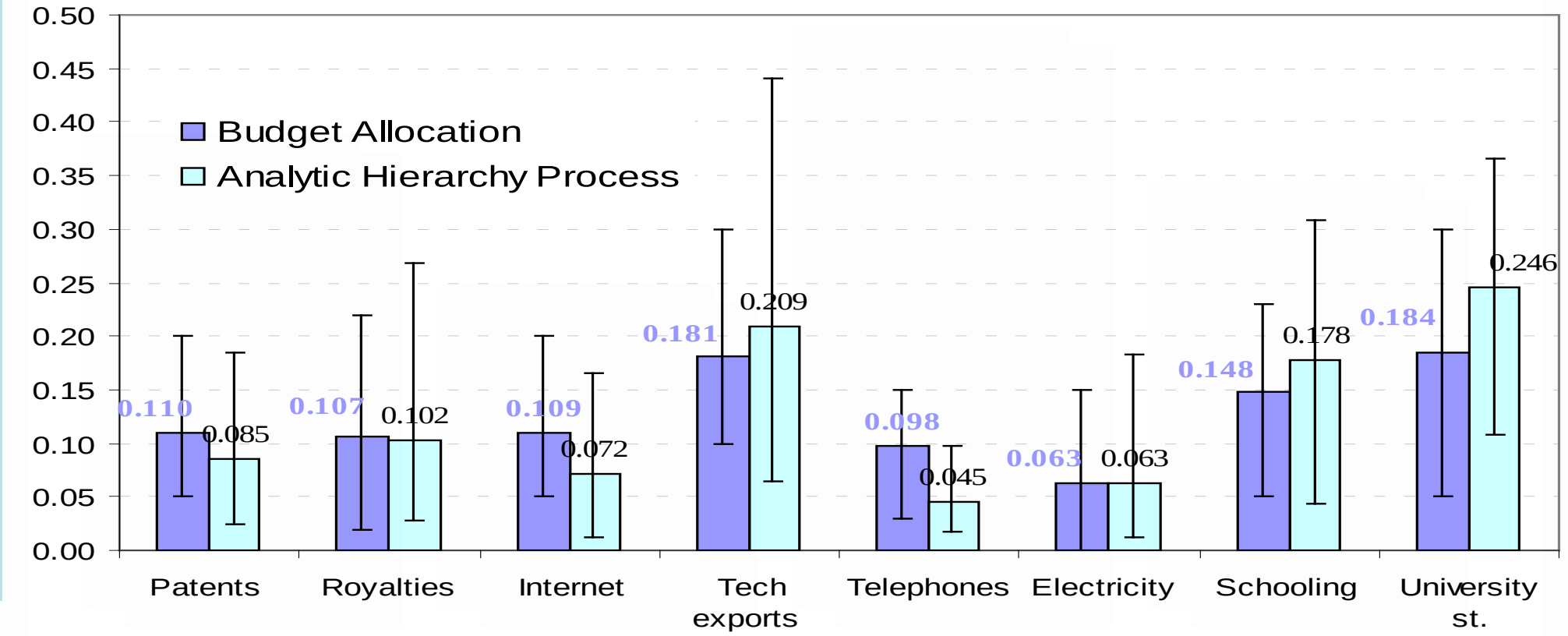


Inconsistencies range from 1.1 % - 45.5%

(desired < 10-20 %)

TAI – Compare BHL and AHP

Weights obtained by Budget allocation are closer to equal weights than those obtained by Analytic Hierarchy Process



AHP – other example

Example 2 : Gender Equality Index

AHP

Purpose : Assign weights at the domain level

Experts: EIGE's Working Group on the Gender Equality Index and EIGE's Expert Forum. Experts'

Response rate 50% Based on **consistency ratio**, 60% of experts weights kept

Conjoint analysis - CA

- BAL and AHP possible when limited numbers of dimensions/indicators
- Alternatively, expert-based weights can be derived from **conjoint analysis (CA)**
  - Respondents rank "alternative scenarios" (Hair *et al.*, 1995)
    - Each scenario → different values of the indicators/dimensions
  - Approach frequently used in **marketing** and **consumer** research
  - **Decompositional** multivariate data analysis.

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